

Determining the number of factors: the example of the NEO-PI-R

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The number of factors to extract from a data set is a long time problem in psychometrics. Henry Kaiser described it as easy and one that he solved everyday before breakfast. But the problem was to find the right solution (Horn and Engstrom, 1979). Here I examine the number of factors to extract from a well defined data set that supposedly has a "known" answer. Unfortunately, each of four different procedures gives a different answer.

The correlation matrix examined is that of the 30 facets of the NEO-PI-R, taken from the test manual of the NEO. (The matrix was kindly supplied by Suitbert Ertel to the readers of the IDANET newsgroup.) The number of cases is 1,000.

1 Parallel analysis and the Scree Test

One of the most commonly used procedures is simply to plot the eigen values of the first N principal components and look for sharp breaks in the plot. Cattell's scree test is analogically finding where a mountain starts after climbing up the pile of rubble (the scree) that has fallen off (Cattell, 1966).

Parallel Analysis is an alternative to the scree test and compares the eigen values of the observed data to those of a random data set of the same size.

Both of these tests are implemented in the `fa.parallel` function in the *psych* package in R.

The solution shown in Figure 1 suggests that five is the most appropriate number of factors. This is, of course, what one would expect from the definitive measure for the "Five Factor Theory".

2 Chi Square goodness of fit estimates

Some advocate that factors should be extracted until the correlations in the residual matrix do not differ from what would be expected by chance. Using Maximum Likelihood estimates of the factor structure, we can apply either Bartlett's test or the Lawley and Maxwell test

Parallel Analysis Scree Plots

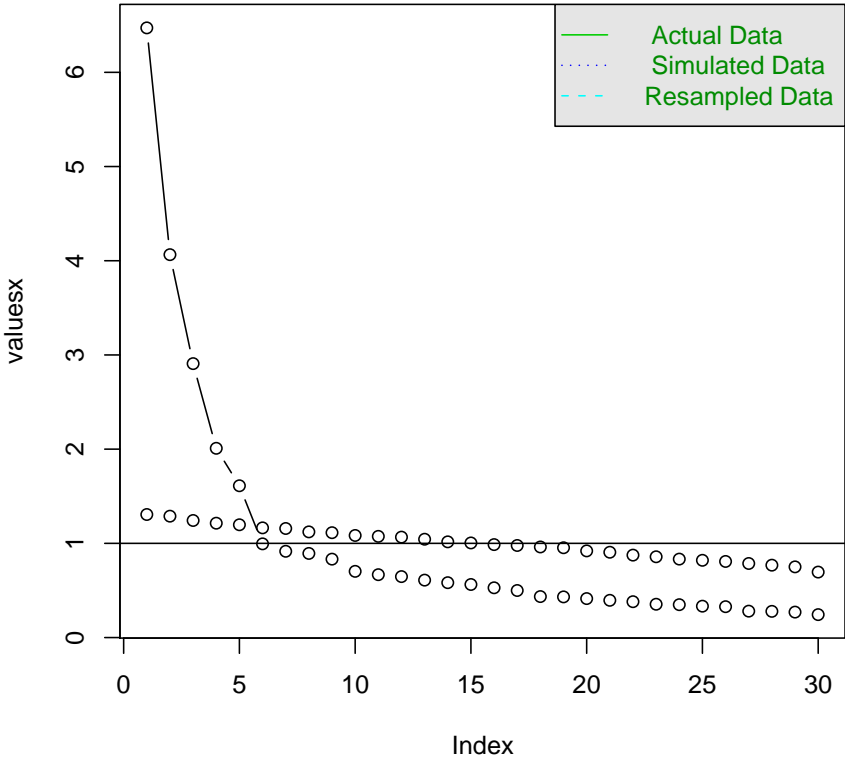


Figure 1: The scree test and parallel analyses of the NEO-PI-R suggest that five factors are most appropriate. This is, of course, the expected solution.

and then show the χ^2 values as a function of the number of factors extracted. Unfortunately, this procedure leads to the interesting phenomena of extracting more factors the larger the sample size. For this data set, with 1,000 subjects, the χ^2 show that the residuals are greater than a 0 matrix for up to 15 factors.

Table 1: The χ^2 test for the number of factors tests the probability that the residual matrix does not differ from a chance matrix. Being very sensitive to sample size, this test will tend to overfactor for large samples.

```
> vss.neo16.mle <- VSS(neo.df, 16, pc = "mle", n.obs = 1000)
> round(vss.neo16.mle[, 1:3], 2)
```

	dof	chisq	prob
1	405	7996.56	0.00
2	376	5517.28	0.00
3	348	3514.73	0.00
4	321	2113.94	0.00
5	295	1443.45	0.00
6	270	1150.59	0.00
7	246	897.84	0.00
8	223	709.15	0.00
9	201	579.26	0.00
10	180	486.38	0.00
11	160	387.71	0.00
12	141	310.90	0.00
13	123	233.19	0.00
14	106	168.11	0.00
15	90	123.64	0.01
16	75	89.96	0.11

3 Very Simple Structure

Yet another way to choose the number of factors is to use the Very Simple Structure (VSS) criterion (Revelle and Rocklin, 1979). VSS recognizes that most users of factor analysis tend to interpret factor output by focusing their attention on the largest loadings for every variable and ignoring the smaller ones. (And in fact score the scales based not upon factor loading estimates but rather just unit weighted composites of these salient variables.) Very Simple Structure operationalizes this tendency by comparing the original correlation matrix to that reproduced by a simplified version (S_{ck}) of the original factor matrix (F). S_{ck} is

composed of just the c greatest (in absolute value) loadings for each variable and k is the number of factors. C (or complexity) is a parameter of the model and may vary from 1 to the number of factors. Essentially this is a simple way of doing confirmatory factor analysis.

The basic factor model is

$$R \approx FF' + U^2 \quad (1)$$

where U^2 is a diagonal matrix of uniquenesses.

The Very Simple Structure (VSS) model is

$$R \approx SS' + U^2 \quad (2)$$

with residuals

$$R^* = R - (SS' + U^2) \quad (3)$$

and the VSS criterion is

$$VSS_{ck} = 1 - \frac{\sum_{i=1}^n \sum_{j=1}^n r_{ij}^{*2}}{\sum_{j=1}^n \sum_{i=1}^n r_{ij}^2} \quad (4)$$

VSS for a given complexity will tend to peak at the optimal (most interpretable) number of factors (Revelle and Rocklin, 1979). Simulations have shown that if the data are simple structured, and each variable is of complexity one (i.e., has only one non-zero loading), then either under or over factoring will lead to a worse solution than extracting the correct number of factors. Many personality scales are somewhat more complex in that some items are of complexity two (e.g., Hofstee et al. (1992)). In this case, the VSS criterion for complexity two will indicate the optimal number of interpretable factors.

4 Hierarchical Cluster Analysis and the Internal Structure of Tests

An alternative to factor analysis for examining the internal structure of tests and to determine the optimal number of scales to derive from a multidimensional battery is hierarchical cluster analysis using the ICLUST algorithm (Revelle, 1979). Scales are combined until β , the worst split half reliability coefficient, fails to increase. Revelle (1979) suggested that β is an estimate of the general factor saturation of a test. Although Zinbarg et al. (2005) showed that McDonald's hierarchical ω is a better estimate of the general factor saturation, it is still the case that using the β coefficient as a stopping criterion in hierarchical clustering produces very interpretable scales. ICLUST is implemented in the `ICLUST` and `ICLUST.rgraph` functions in the *psych* package in R.

Two of these four clusters are moderately intercorrelated ($r = .41$) suggesting that a three cluster solution might be better (Table 2, Figure 4).

```
> vss.neo.mle <- VSS(neo.df, pc = "mle", n.obs = 1000)
> VSS.plot(vss.neo.mle)
```

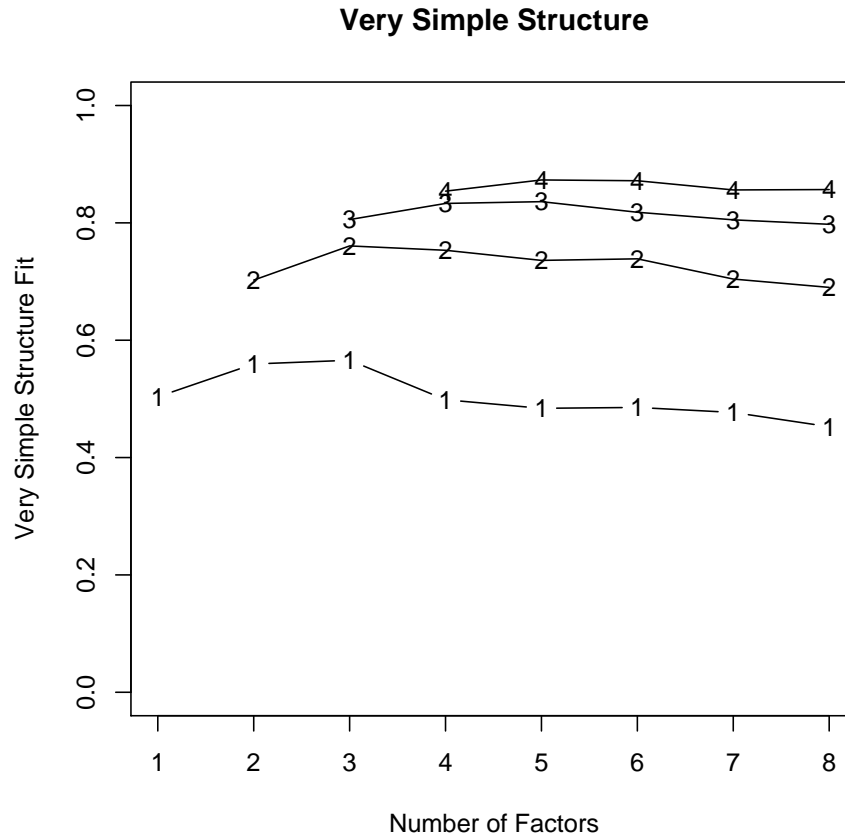


Figure 2: The Very Simple Structure criterion applied to the 30 facets of the NEO-PI-R. The most interpretable number of factors of complexity 1 or 2 is a 3 factor solution.

```

> neo.clus <- ICLUST(neo.df)
> ICLUST.rgraph(neo.clus, labels = neo.label[, ], min.size = 4)

```

ICLUST

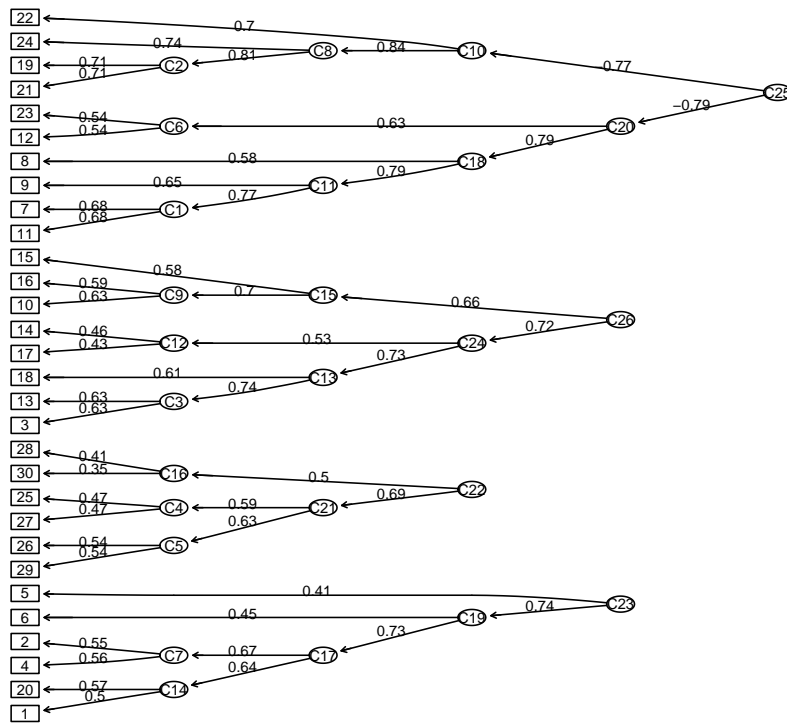


Figure 3: ICLUST analysis of the NEO

Table 2: The four cluster solution suggests that two of them are moderately intercorrelated.

```
> neo.clus$cor
```

	C22	C26	C23	C25
C22	1.00	0.41	-0.01	-0.02
C26	0.41	1.00	0.07	0.28
C23	-0.01	0.07	1.00	0.28
C25	-0.02	0.28	0.28	1.00

```
> neo.clus$cor
```

	C22	C26	C23	C25
C22	1.00	0.41	-0.01	-0.02
C26	0.41	1.00	0.07	0.28
C23	-0.01	0.07	1.00	0.28
C25	-0.02	0.28	0.28	1.00

```
> neo.clus <- ICLUST(neo.df, nclusters = 3, alpha = 0, beta = 0)
```

Cluster solutions are useful because they in fact represent the effect of unit weighted scoring. That is, while a five factor solution is frequently discussed, when actually forming unit weighted composites of items or of facets, what is measured is not five independent factors but rather five highly intercorrelated scales. Using the simple concept that scales should be combined as long as the general factor (estimated in ICLUST by coefficient β) of the higher level scale is greater than that of either subcomponent, a three or four cluster solution is most appropriate.

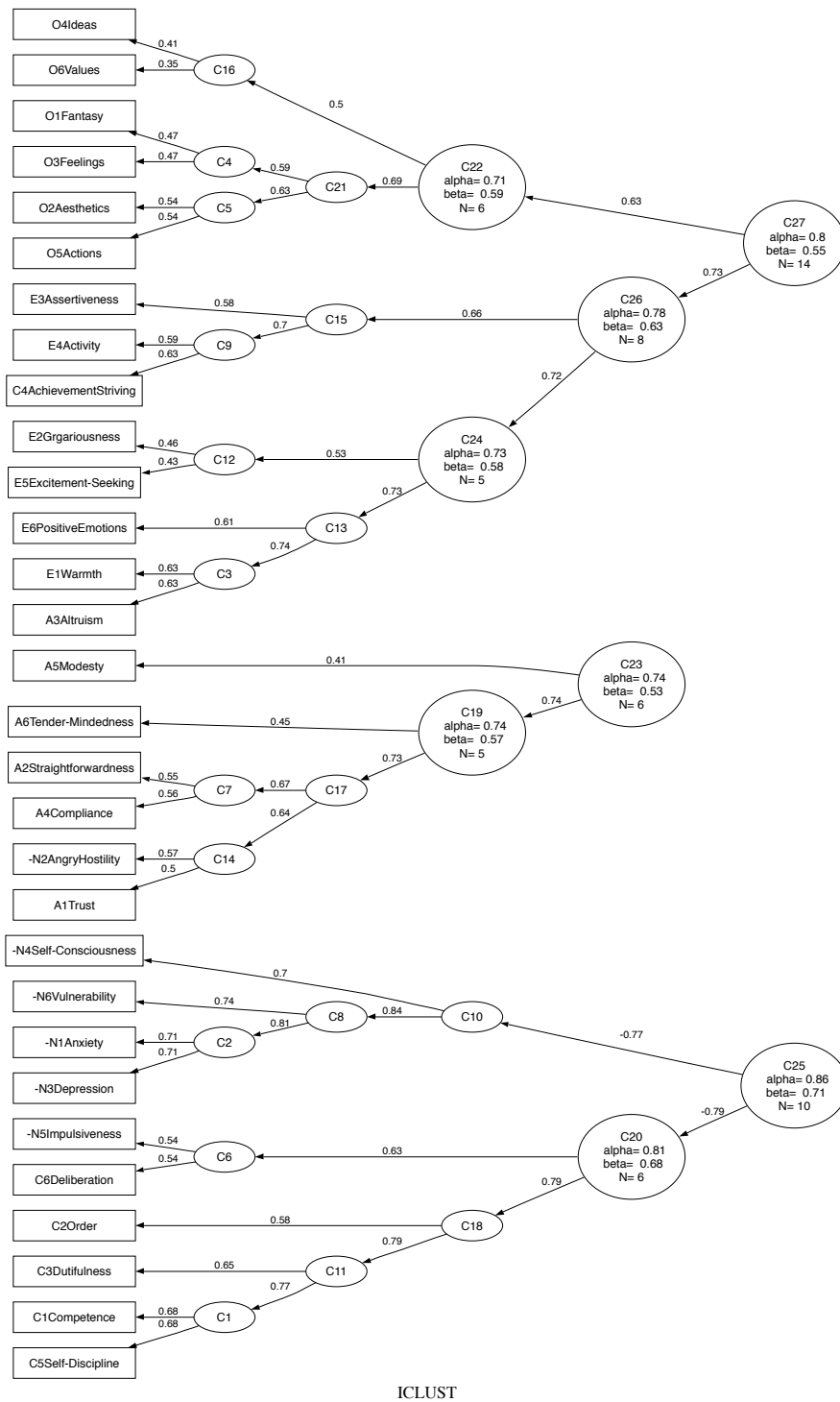


Figure 4: ICLUST analysis of the NEO set to extract 3 clusters

5 Interpreting the factors

Yet a final way to decide on the number of factors is factor interpretability. Here we compare 3, 4, and 5 factor solutions with a VARIMAX rotation to simple structure and the 5 factor solution with a oblimin transformation (using the *GPArotation* package).

Table 3: 3 and 4 factor solutions to the NEO PI- R

```
> neo3 <- factanal(covmat = neo, factors = 3)
> neo4 <- factanal(covmat = neo, factors = 4)
> neo34 <- cbind(neo3$loadings, neo4$loadings)
> round(neo34, 2)
```

	Factor1	Factor2	Factor3	Factor1	Factor2	Factor3	Factor4
N1Anxiety	-0.66	0.05	0.00	0.00	0.77	-0.11	-0.01
N2AngryHostility	-0.53	0.06	-0.46	0.00	0.59	-0.10	-0.49
N3Depression	-0.77	-0.05	-0.04	-0.08	0.78	-0.28	-0.05
N4Self-Consciousness	-0.64	-0.17	0.02	-0.20	0.68	-0.18	0.02
N5Impulsiveness	-0.54	0.29	-0.17	0.28	0.44	-0.28	-0.19
N6Vulnerability	-0.77	-0.18	0.02	-0.18	0.68	-0.39	0.02
E1Warmth	0.16	0.60	0.44	0.62	-0.10	0.14	0.41
E2Gregariousness	0.11	0.49	0.14	0.51	-0.12	0.04	0.12
E3Assertiveness	0.42	0.53	-0.24	0.50	-0.30	0.31	-0.27
E4Activity	0.21	0.57	-0.16	0.51	0.03	0.41	-0.21
E5Excitement-Seeking	-0.04	0.51	-0.28	0.49	0.02	-0.01	-0.30
E6PositiveEmotions	0.09	0.68	0.18	0.68	-0.04	0.12	0.14
O1Fantasy	-0.32	0.39	-0.14	0.42	0.15	-0.31	-0.15
O2Aesthetics	-0.04	0.37	0.13	0.38	0.06	0.02	0.10
O3Feelings	-0.21	0.61	0.02	0.58	0.29	0.05	-0.02
O4Ideas	0.10	0.39	0.03	0.42	-0.16	-0.03	0.02
O5Actions	0.17	0.36	-0.10	0.37	-0.17	0.06	-0.11
O6Values	-0.01	0.26	-0.07	0.29	-0.09	-0.12	-0.08
A1Trust	0.28	0.17	0.51	0.23	-0.31	0.06	0.51
A2Straightforwardness	0.17	-0.21	0.60	-0.20	-0.04	0.22	0.61
A3Altruism	0.22	0.37	0.59	0.37	-0.05	0.30	0.57
A4Compliance	0.13	-0.17	0.71	-0.11	-0.14	0.02	0.72
A5Modesty	-0.17	-0.24	0.50	-0.22	0.16	-0.07	0.50
A6Tender-Mindedness	-0.02	0.20	0.56	0.23	0.03	0.01	0.55
C1Competence	0.69	0.25	0.07	0.21	-0.40	0.61	0.06
C2Order	0.43	0.04	0.06	-0.05	-0.07	0.64	0.04
C3Dutifulness	0.54	0.03	0.28	-0.03	-0.22	0.61	0.28
C4AchievementStriving	0.48	0.35	-0.06	0.27	-0.10	0.69	-0.10
C5Self-Discipline	0.69	0.16	0.10	0.08	-0.32	0.76	0.08
C6Deliberation	0.49	-0.18	0.20	-0.23	-0.24	0.47	0.21

Table 4: Five factor solution to the NEO-PI-R.

```
> neo5 <- factanal(covmat = neo, factors = 5)
```

Call:

```
factanal(factors = 5, covmat = neo)
```

Uniquenesses:

	N1Anxiety	N2AngryHostility	N3Depression	N4Self-Consciousness
	0.399	0.405	0.304	0.471
	N6Vulnerability	E1Warmth	E2Gregariousness	E3Assertiveness
	0.361	0.405	0.674	0.501
	E5Excitement-Seeking	E6PositiveEmotions	O1Fantasy	O2Aesthetics
	0.621	0.448	0.593	0.516
	O4Ideas	O5Actions	O6Values	A1Trust A2S
	0.722	0.487	0.847	0.591
	A3Altruism	A4Compliance	A5Modesty	A6Tender-Mindedness
	0.388	0.442	0.677	0.644
	C2Order	C3Dutifulness	C4AchievementStriving	C5Self-Discipline
	0.586	0.483	0.413	0.311

Loadings:

	Factor1	Factor2	Factor3	Factor4	Factor5
N1Anxiety	0.766	-0.116			
N2AngryHostility	0.594	-0.100	-0.478		
N3Depression	0.783	-0.266		-0.106	
N4Self-Consciousness	0.676	-0.179		-0.169	
N5Impulsiveness	0.431	-0.310	-0.234	0.281	
N6Vulnerability	0.677	-0.383		-0.158	
E1Warmth	-0.116	0.116	0.324	0.648	0.205
E2Gregariousness	-0.142			0.543	
E3Assertiveness	-0.300	0.299	-0.318	0.399	0.246
E4Activity		0.388	-0.278	0.479	0.170
E5Excitement-Seeking			-0.378	0.464	0.141
E6PositiveEmotions				0.706	0.203
O1Fantasy	0.161	-0.283	-0.151	0.170	0.499
O2Aesthetics			0.143		0.664
O3Feelings	0.304			0.402	0.463
O4Ideas	-0.155			0.219	0.454
O5Actions	-0.152	0.127			0.683
O6Values				0.112	0.343
A1Trust	-0.310		0.486	0.235	0.134
A2Straightforwardness		0.222	0.623		-0.113
A3Altruism		0.255	0.501	0.538	
A4Compliance	-0.140		0.732		
A5Modesty	0.156		0.509		-0.162
A6Tender-Mindedness			0.520	0.267	0.115
C1Competence	-0.399	0.614		0.181	0.117
C2Order		0.619			-0.146
C3Dutifulness	-0.209	0.626	0.285		
C4AchievementStriving		0.699	-0.126	0.213	0.169

Table 5: Oblique transformations of the five factor solution indicate that factors one and two are highly correlated and factors four and five are moderately correlated.

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	1.00	-0.41	-0.12	-0.13	-0.07
[2,]	-0.41	1.00	0.07	0.21	0.08
[3,]	-0.12	0.07	1.00	0.01	-0.16
[4,]	-0.13	0.21	0.01	1.00	0.32
[5,]	-0.07	0.08	-0.16	0.32	1.00

	Factor1	Factor2	Factor3	Factor4	Factor5
N1Anxiety	0.80	0.04	0.00	0.07	0.00
N2AngryHostility	0.58	0.05	-0.44	-0.12	0.01
N3Depression	0.78	-0.11	0.02	-0.04	0.05
N4Self-Consciousness	0.69	-0.03	0.10	-0.09	-0.05
N5Impulsiveness	0.37	-0.25	-0.28	0.30	-0.02
N6Vulnerability	0.65	-0.25	0.09	-0.06	-0.05
E1Warmth	-0.08	0.03	0.17	0.69	0.10
E2Gregariousness	-0.15	-0.07	-0.09	0.55	-0.01
E3Assertiveness	-0.29	0.23	-0.41	0.25	0.16
E4Activity	0.08	0.39	-0.38	0.36	0.09
E5Excitement-Seeking	-0.04	-0.07	-0.48	0.38	0.03
E6PositiveEmotions	-0.05	0.01	-0.12	0.70	0.08
O1Fantasy	0.09	-0.32	-0.14	0.11	0.47
O2Aesthetics	0.13	0.05	0.18	-0.01	0.70
O3Feelings	0.33	0.08	-0.10	0.34	0.42
O4Ideas	-0.17	-0.09	-0.01	0.15	0.43
O5Actions	-0.15	0.05	-0.03	-0.13	0.71
O6Values	-0.12	-0.16	-0.08	0.06	0.32
A1Trust	-0.28	-0.05	0.42	0.31	0.11
A2Straightforwardness	0.06	0.22	0.63	0.03	-0.06
A3Altruism	0.02	0.21	0.35	0.63	-0.11
A4Compliance	-0.08	-0.03	0.73	0.11	0.03
A5Modesty	0.20	-0.05	0.52	0.06	-0.12
A6Tender-Mindedness	0.07	-0.03	0.46	0.37	0.10
C1Competence	-0.29	0.56	-0.02	0.08	0.10
C2Order	0.06	0.66	0.00	0.00	-0.13
C3Dutifulness	-0.06	0.62	0.27	-0.03	0.05
C4AchievementStriving	0.04	0.72	-0.17	0.07	0.16
C5Self-Discipline	-0.18	0.73	-0.01	0.08	-0.08
C6Deliberation	-0.12	0.50	0.28	-0.23	0.02

6 Comparing solutions

It is possible to compare solutions by examining the factor congruence of the factor/cluster loadings.

```
> round(factor.congruence(neo4, neo5), 2)
```

	Factor1	Factor2	Factor3	Factor4	Factor5
Factor1	-0.16	0.22	-0.12	0.93	0.77
Factor2	1.00	-0.56	-0.22	-0.18	-0.10
Factor3	-0.56	1.00	0.19	0.32	0.04
Factor4	-0.24	0.22	0.99	0.15	-0.09

```
> round(factor.congruence(neo3, neo4), 2)
```

	Factor1	Factor2	Factor3	Factor4
Factor1	0.19	-0.91	0.84	0.27
Factor2	0.99	-0.13	0.30	-0.05
Factor3	0.04	-0.23	0.26	1.00

```
> round(factor.congruence(neo3, neo5), 2)
```

	Factor1	Factor2	Factor3	Factor4	Factor5
Factor1	-0.92	0.85	0.24	0.26	0.07
Factor2	-0.14	0.29	-0.16	0.93	0.75
Factor3	-0.24	0.25	0.98	0.21	-0.05

When comparing the 4 and 5 factor solutions, three are practically identical and the first factor of the 4 factor solution is a blend of the fourth and fifth factors. One factor (three) of the three factor solution corresponds with the third of of the five factor solution, and factor one of the three is a blend of factors one and two of the five and factor two is a blend of four and five.

What are these three factors?

Factor one is a combination of Conscientiousness and Emotional stability, perhaps interpreted as emotional and impulse control. Factor two of Extraversion and Openness reflects a general positive excitement to people and ideas, while factor three is the general agreeableness factor of trust in others and generally being nice.

References

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