

Contents

Part I Background to Psychological Measurement

1	Introduction	5
1.1	Constructs and measures (Chapter 2)	6
1.1.1	Observational and Experimental Psychology	8
1.1.2	Data = Model + Residual	9
1.2	A theory of data (Chapter 2)	9
1.3	Basic summary statistics – problems of scale (Chapter 3)	10
1.4	Covariance, regression, and correlation (Chapter 4)	10
1.5	Multiple and partial correlation and regression (Chapter 5)	10
1.6	Factor, Principal Components and Cluster Analysis (Chapter 6)	10
1.7	Classical theory and the Measurement of Reliability (Chapter 7)	11
1.8	Latent Trait Theory - The “New Psychometrics” (Chapter 8)	12
1.9	Validity (Chapter 9)	12
1.9.1	Decision Theory	12
1.10	Structural Equation Modeling (Chapter 11)	13
1.10.1	Appendix – Basic R	13
1.10.2	Appendix - Review of Matrix algebra	13
1.11	General comments	13
2	A Theory of Data	15
2.1	A Theory of Data: Objects, People, and Comparisons	15
2.1.1	Modeling the comparison process	16
2.2	Models and model fitting	16
2.3	A brief diversion: Functions in R	18
2.4	Tournaments: Ordering people ($p_i > p_j$)	19
2.4.1	Scaling of People	20
2.4.2	Alternative approaches to scaling people	22
2.4.3	Assigning numbers to people – the problem of rewarding merit	23
2.5	Social Networks: Proximities of People ($ p_i - p_j < \delta$)	23
2.5.1	Rectangular data arrays of similarity	23
2.5.2	Square arrays of similarity	24
2.6	The Scaling of Objects ($o_i < o_j$)	25
2.6.1	Weber-Fechner scales of subjective experience	27

2.6.2	Thurstonian Scaling	28
2.6.3	Alternative solutions to the ranking of objects	32
2.6.4	Why emphasize Thurstonian scaling?	34
2.7	Multiple Dimensional Scaling: Distances between Objects ($ o_i - o_j < o_k - o_l $)	34
2.8	Preferential Choice: Unfolding Theory ($ s_i - o_j < s_k - o_l $)	36
2.8.1	Individual Preferences – the I scale	36
2.8.2	Joint Preferences – the J scale	38
2.8.3	Partially ordered metrics	38
2.8.4	Multidimensional Unfolding	39
2.9	Measurement of Attitudes and Abilities (comparing s_i, o_j)	40
2.9.1	Measurement of abilities ($s_i > o_j$)	40
2.9.2	Measurement of attitudes ($ s_i - o_j < \delta$)	43
2.10	Theory of Data: some final comments	45
3	The problem of scale	47
3.1	Four broad classes of scales	49
3.1.1	Factor levels as Nominal values	51
3.1.2	Integers and Reals: Ordinal or Metric values?	51
3.2	Graphical and numeric summaries of the data	51
3.2.1	Sorting data as a summary technique	52
3.3	Numerical estimates of central tendency	52
3.3.1	Mode: the most frequent	53
3.3.2	Median: the middle observation	55
3.3.3	3 forms of the mean	57
3.3.4	Comparing variables or groups by their central tendency	59
3.4	The effect of non-linearity on estimates of central tendency	59
3.4.1	Circular Means	60
3.5	Whose mean? The problem of point of view	62
3.5.1	Average length of time in psychotherapy	62
3.5.2	Average class size	63
3.6	Non-linearity and interpretation of experimental effects	63
3.6.1	Linearity, non-linearity and the properties of measurement	66
3.7	Measures of dispersion	66
3.7.1	Measures of range	66
3.7.2	Average distance from the central tendency	67
3.7.3	Deviation scores and the standard deviation	68
3.7.4	Coefficient of variation	69
3.8	Geometric interpretations of Variance and Covariance	70
3.9	Variance, Covariance, and Distance	71
3.10	Standard scores as unit free measures	72
3.11	Assessing the higher order moments of the normal and other distributions	72
3.12	Generating commonly observed distributions	77
3.13	Mixed distributions	78
3.14	Robust measures of dispersion	79
3.15	Monotonic transformations of data and “Tukey’s ladder”	81
3.16	What is the fundamental scale?	81

4	Covariance, Regression, and Correlation	85
4.1	Correlation as the geometric mean of regressions	88
4.2	Regression and prediction	90
4.3	A geometric interpretation of covariance and correlation	91
4.4	The bivariate normal distribution	93
4.4.1	Confidence intervals of correlations	93
4.4.2	Testing whether correlations differ from zero	94
4.4.3	Testing the difference between correlations	95
4.5	Other estimates of association	98
4.5.1	Pearson correlation equivalents	99
4.6	Other measures of association	106
4.6.1	Naturally dichotomous data	108
4.6.2	Measures of association for categorical data	112
4.6.3	Intraclass Correlation	113
4.6.4	Quantile Regression	114
4.6.5	Kendall's Tau	115
4.6.6	Circular-circular and circular-linear correlations	115
4.7	Alternative estimates of effect size	117
4.8	Sources of confusion	118
4.8.1	Restriction of range	118
4.8.2	Spurious correlations	119
4.8.3	Non linearity, outliers and other problems: the importance of graphics	123
5	Multiple correlation and multiple regression	125
5.1	The variance of composites	125
5.2	Multiple regression	126
5.2.1	Direct and indirect effects, suppression and other surprises	128
5.2.2	Interactions and product terms: the need to center the data	128
5.2.3	Confidence intervals of the regression and regression weights	130
5.2.4	Multiple regression from the covariance/correlation matrix	132
5.2.5	The robust beauty of linear models	132
5.3	Partial and semi-partial correlation	132
5.3.1	Alternative interpretations of the partial correlation	134
5.4	Alternative regression techniques	134
5.4.1	Logistic regression	134
5.4.2	Poisson regression, quasi-Poisson regression, and negative-binomial regression	135
5.4.3	Using multiple regression for circular data	137
5.4.4	Robust regression using M estimators	139

Part II Latent Variable Models

6	Constructs, Components, and Factor models	143
6.1	Principal Components: an observed variable model	145
6.1.1	Eigenvalues and Eigenvectors	145
6.1.2	Principal components	147
6.2	Exploratory Factor Analysis: a latent variable model	148

6.2.1	Principal Axes Factor Analysis as an eigenvalue decomposition of a reduced matrix	150
6.2.2	Maximum Likelihood Factor Analysis and its alternatives	151
6.2.3	Comparing extraction techniques	154
6.2.4	Exploratory analysis with more than one factor/component	154
6.2.5	Comparing factors and components- part 1	155
6.3	Rotations and Transformations	158
6.3.1	Orthogonal rotations	162
6.3.2	Oblique transformations	162
6.3.3	Non-Simple Structure Solutions: The Simplex and Circumplex	166
6.3.4	Hierarchical and higher order models	167
6.3.5	Comparing factor solutions	172
6.4	The number of factors/components problem	172
6.5	The number of subjects problem	177
6.6	Confirmatory Factor Analysis	177
6.7	Alternative procedures for reducing the complexity of the data	178
6.7.1	MDS solutions remove the general factor	180
6.7.2	Cluster analysis – poor man’s factor analysis?	181
6.8	Estimating factor scores, finding component and cluster scores	190
6.8.1	Comparing factors and components – part 2	191

Part III Classical and Modern Test Theory

7	Classical Test Theory and the Measurement of Reliability	195
7.1	Reliability and True Scores	197
7.1.1	Parallel Tests, Reliability, and Corrections for Attenuation	198
7.1.2	Tau equivalent and congeneric tests	201
7.2	Reliability and internal structure	203
7.2.1	Split half reliability	203
7.2.2	Domain sampling	205
7.2.3	The internal structure of a test. Part 1: coefficient α	207
7.2.4	The internal structure of a test. Part 2: Guttman’s lower bounds of reliability	209
7.2.5	The internal structure of a test. Part 3: coefficients α , β , ω_h and ω_t	214
7.3	A comparison of internal consistency estimates of reliability	217
7.4	Estimation of reliability	220
7.4.1	Test-retest reliability: Stability across time	221
7.4.2	Intraclass correlations and the reliability of ratings across judges	222
7.4.3	Generalizability theory: reliability over facets	226
7.4.4	Reliability of a composite test	226
7.4.5	Reliability of a difference score	227
7.5	Using reliability to estimate true scores	227
8	The “New Psychometrics” – Item Response Theory	231
8.1	Monotonic trace lines: the measurement of ability	232
8.1.1	Rasch Modeling - one parameter IRT	232
8.1.2	The normal ogive – another one parameter models	237
8.1.3	Parameter estimation	238

8.1.4	Item information	239
8.1.5	Two parameter models	240
8.1.6	Three parameter models	241
8.1.7	Four parameter models	241
8.2	Polytomous items	243
8.2.1	Ordered response categories	244
8.2.2	Multiple choice ability items	245
8.3	IRT and factor analysis of items	247
8.4	Test bias and Differential Item Functioning	247
8.5	Non-monotone trace lines – the measurement of attitudes	247
8.5.1	Unfolding theory revisited	247
8.5.2	Political choice	247
8.6	Item banking and item comparison	248
8.7	IRT and adaptive testing	248
8.8	Classical versus IRT models – does it make a difference?	248
9	Validity	249
9.1	Types of validity	249
9.1.1	Face or Faith	249
9.1.2	Concurrent and Predictive	249
9.1.3	Construct	249
9.2	Validity and cross validation	250
9.2.1	Cross validation	250
9.2.2	Resampling and cross validation	250
9.3	Validity: an modern perspective	250
9.4	Validity for what?	250
9.4.1	Validity for the institution	250
9.4.2	validity for the individual	250
9.5	Validity and decision making	250
10	Simulating latent variable models	251
10.1	Simulate one latent variable and n observed variables	251
10.1.1	generate the data in a function	253
10.2	SEM for one sample from the population	254
10.2.1	a smaller sample	256
10.2.2	multiple simulations - varying sample size	257
10.3	Statistics for a more complicated model	259
10.3.1	Fitting a 1 factor model to two factor data	260
10.3.2	fitting a 2 factor model to two factor data	262
10.3.3	A set of simulations for the two factor model	264
11	Reliability + Validity = Structural Equation Models	265
11.1	Generating simulated data structures	265
11.2	Measures of fit	265
11.2.1	χ^2	265
11.2.2	GFI, NFI, ...	266
11.2.3	RMSEA	266
11.3	Reliability (Measurement) models	266

11.3.1	One factor — congeneric measurement model	266
11.3.2	Two (perhaps correlated) factors	266
11.3.3	Hierarchical measurement models	266
11.4	Reliability + Validity = Structural Equation Models	266
11.4.1	Factorial invariance	266
11.4.2	Multiple group models	266
11.5	Evaluating goodness of fit	266
11.5.1	Model misspecification: Item quality	266
11.5.2	Model misspecification: failure to include variables	267
11.5.3	Model misspecification: incorrect structure	267
11.6	What does it mean to fit a model	267
12	Testing alternative models of data	269
12.1	One factor — congeneric data model	269
12.1.1	Generating the data	269
12.1.2	Estimate a congeneric model	272
12.1.3	Estimate a tau equivalent model with equal true score and unequal error loadings	273
12.1.4	Estimate a parallel test model with equal true score and equal error loadings	274
12.1.5	Estimate a parallel test model with fixed loadings	276
12.1.6	Comparison of models	277
12.2	Two (perhaps correlated) factors	277
12.2.1	Generating the data	277
12.2.2	Exploratory Factor analysis of the data	279
12.2.3	Confirmatory analysis with a predicted structure	281
12.2.4	Confirmatory factor analysis with two independent factors with equal loadings within factors	283
12.2.5	Structure invariance, part I— unequal loadings within factors - matched across factors	284
12.2.6	Estimate two correlated factors	286
12.3	Hierarchical models	293
12.3.1	Two Correlated factors with a g factor	293
12.3.2	Generating the data for 3 correlated factors	296
12.3.3	Exploratory factor analysis with 3 factors	297
12.3.4	Three correlated factors with a g factor	299
12.3.5	Bifactor solutions	301
12.3.6	Schmid Leiman transformations to orthogonalize the factors	304
12.3.7	Omega as an estimate of reliability	306
13	sem in R and in LISREL	307
13.1	Example data set 1: 9 cognitive variables (from Rakov and Marcoulides)	307
13.2	Using R to analyze the data set	309
13.2.1	An initial formulation is empirically underidentified	309
13.2.2	Adjusting to model to converge	310
13.2.3	Modifying the model to improve the fit	312
13.2.4	Changing from a regression model to a correlation model	314
13.3	Using LISREL to analyze the data set	316

13.3.1	Instructions for using the SSCC	318
13.3.2	Modify the model to allow for correlated errors	325
13.4	Comparing the R and LISREL output	333
13.5	Testing for factorial invariance	333
13.5.1	Testing for factorial equivalence in multiple groups	335
14	Further issues: Item quality	337
14.1	Continuous, ordinal, and dichotomous data	337
14.2	Simple structure versus circumplex structure	337
14.3	Data generation using the circ.sim function	338
14.4	Simple structure - normal items	338
14.4.1	5 categories of responses	343
14.4.2	3 categories of responses	345
14.4.3	dichotomous items	351
14.5	Circumplex structure - normal items	355
14.5.1	Fitting a circumplex data set with a simple structure model	357
14.5.2	An alternative model	360
14.6	Simple Structure - categorical and skewed items	363
14.6.1	Two dimensions with 4 point scales, differing in skew	363
14.6.2	An alternative model of two bipolar dimensions	368
14.7	Forming clusters or homogeneous item composites	373
15	Evaluating models	379
15.1	Model misspecification: failure to include variables	379
15.1.1	Misspecified Linear Regression	380
15.1.2	Regression with the correct variables included	381
15.1.3	Misspecified Structural Equation Models	383
15.1.4	Three predictors - alternative models	387
15.1.5	Three predictors, model the correlations with one latent variable	393
15.1.6	Three predictors with shared “error”	395
15.1.7	Reverse the causal paths	399
15.2	Including the correct variables, but misspecifying the models	400
15.2.1	Including the correct variables in linear regression	401
15.2.2	Including the correct variables in the Structural Equation	403
15.2.3	Direct the causal path	406
15.3	Measures of fit	412
15.3.1	χ^2	412
15.3.2	GFI, NFI, ...	412
15.3.3	RMSEA	412
15.4	What does it mean to fit a model	412
16	Multidimensional Scaling and Multi-Mode Methods	413
16.1	Basic models of Multidimensional Scaling	413
16.1.1	Metric models	413
16.1.2	Non-metric models	413
16.2	Measuring Individual Differences in MDS	413
16.2.1	INDSCAL and ALSCAL	413

Part IV The construction of tests and the analysis of data

17 Basic issues in scale construction– An example	417
17.1 Steps towards scale construction	417
17.2 Scale analysis: part 1: analyzing the dimensionality of the items	417
17.3 Preliminary steps	418
17.3.1 Data checking	418
17.4 Data reduction	419
17.4.1 Background: eigen values and eigen vectors	419
17.4.2 Principal Components	419
17.4.3 Factor analysis	420
17.4.4 Estimating the number of components or factors to extract	420
17.5 Factor analysis of the data set	422
17.5.1 Principal Components	422
17.5.2 Factor Analysis	423
17.5.3 Simple Structure	424
17.5.4 Cluster analysis as an alternative structural model	425
17.6 Forming item composites	425
17.7 Evaluating the scale reliabilities	425
17.8 Validating the instrument	425

Part V Appendices

A R: Getting started	429
A.1 R: A statistical programming environment	429
A.2 General comments	430
A.3 Using R in 12 simple steps	431
A.4 Getting started	432
A.4.1 Installing R on your computer	432
A.4.2 Packages and Task Views	432
A.4.3 Help and Guidance	433
A.4.4 Package vignettes	433
A.5 Basic R commands and syntax	434
A.5.1 R is just a fancy calculator	434
A.5.2 Data structures	435
A.6 Entering or getting the data	435
A.7 Basic descriptive statistics	436
A.7.1 Using functions in the psych package	437
A.8 Simple Graphics	438
B R commands	441
B.1 Input and display	441
B.2 moving around	442
B.3 data manipulation	442
B.4 Statistics and transformations	442
B.5 Useful additional commands	443
B.6 Graphics	443

C	R examples	447
C.1	Chapter 2	447
C.1.1	Simulating a chess tournament	447
C.2	Chapter 4	448
D	Particularly useful packages	449
D.1	psych by William Revelle	449
D.2	sem by John Fox	449
D.3	ltm by Dimitris Rizopoulos	449
D.4	GPArotation by Bernaards and Jennrich	449
D.5	robust	449
D.6	nlme: non-linear mixed effects by xxx	449
D.7	psy	449
D.8	multilevel	450
D.9	Rgraphviz	450
D.10	foreign	450
E	Appendix: A Review of Matrices	451
E.1	Vectors	451
E.1.1	Vector multiplication	452
E.1.2	Simple statistics using vectors	453
E.1.3	Combining vectors with cbind and rbind	455
E.2	Matrices	456
E.2.1	Adding or multiplying a vector and a Matrix	457
E.2.2	Matrix multiplication	459
E.2.3	Finding and using the diagonal	462
E.2.4	The Identity Matrix	463
E.2.5	Matrix Inversion	463
E.2.6	Eigenvalues and Eigenvectors	464
E.2.7	Determinants	465
E.3	Matrix operations for data manipulation	465
E.3.1	Matrix operations on the raw data	466
E.3.2	Matrix operations on the correlation matrix	468
E.3.3	Using matrices to find test reliability	468
E.4	Multiple correlation	470
E.4.1	Data level analyses	470
F	More on Matrices	473
F.1	Multiple regression as a system of simultaneous equations	473
F.2	Matrix representation of simultaneous equation	474
F.2.1	Finding the inverse of a 2 x 2 matrix	474
F.3	A numerical example of finding the inverse	476
F.4	Examples of inverse matrices	477
F.4.1	Inverse of an identity matrix	477
F.4.2	The effect of correlation size on the inverse	478

G	Simulation as a tool for research	481
	G.1 Simulating one latent variable and multiple observed	481
	G.2 Simulation of experimental manipulations	481
	G.3 Simulating multiple latent variables with a known structure	481
H	Classics in measurement	483
	H.1 Diameter of the earth	483
	H.2 The period of a pendulum	483
	H.3 Oil, water, and Avogadro's Number	483
	H.4 Measuring reaction time by holding hands	483
	H.5 Dropping torn paper: Measuring the intensity of a nuclear explosion by dropping paper	483
	H.6 Beer cans: Measuring the wave height produced by a nuclear explosion	484
	H.7 Timing waves: Discovering the source of the California summer surf	484
	References	485
	Index	503

List of Figures

1.1	A conceptual overview of the book.	7
2.1	Two comparisons: order and distance	17
2.2	SPLOM of tournament models	21
2.3	Social networks analysis of the data from Table 2.5	26
2.4	Thurstone's model of paired discrimination	29
2.5	Original solution for 11 US cities	36
2.6	Revised MDS solution	37
2.7	Possible I-scales arranged to show ordering of mid-points	39
2.8	The cumulative normal versus logistic	43
2.9	The basic ability model	44
2.10	Basic attitude model.	44
3.1	Experimental and observational research	48
3.2	Box and whiskers	53
3.3	Interpolated quartiles	57
3.4	The effect of non-linearity and variability on estimates of central tendency	61
3.5	Artifacts in scaling	65
3.6	Estimates of dispersion	67
3.7	Three normal curves	73
3.8	z scores and the normal curve	74
3.9	A log normal distribution	76
3.10	Symmetric and single peak does not mean normal	77
3.11	Contaminated normals are difficult to detect	79
3.12	Tukey's ladder of transformations	82
4.1	Galton's data on height	87
4.2	Galton's regression of parental height	88
4.3	pairs.panels	90
4.4	Correlations as cosines	92
4.5	Spearman correlation	102
4.6	The point biserial correlation	103
4.7	Effect of cut points upon phi	105
4.8	The effect of restriction of range on correlation and regression.	119

4.9	The effect of skew on correlatons	122
5.1	Four cases of multiple regression	129
5.2	The (simulated) effect of extraversion and movie induced mood on positive affect.....	131
5.3	Estimating diurnal phase	140
6.1	Factor vs. component models	146
6.2	Component loadings vary by the number of variables	159
6.3	Comparing unrotated and varimax solutions	162
6.4	Simple structure as a path diagram	163
6.5	An oblimin solution to the Harman physical variables problem	165
6.6	A simulation of 24 items showing a circumplex structure	168
6.7	Omega hierarchical uses Schmid Leiman transformation	170
6.8	Parallel analysis of 24 ability tests	174
6.9	Very Simple Structure	175
6.10	Multidimensional scaling removes the general factor	182
6.11	kmeans vs ICLUST	187
6.12	Hierarchical cluster analysis of 24 mental tests	188
7.1	Parallel tests	196
7.2	Reliability	198
7.3	Four congeneric measures	203
7.4	Decomposing test variance	212
7.5	Graphic representation of correlation matrices using <code>cor.plot</code>	218
7.6	One simulated and three real data sets showing hierarchical structure	219
7.7	Reliability of ratings: the ICC	225
7.8	Reliability of differences	227
7.9	Confidence intervals vary with reliability	228
8.1	Rasch scaling	235
8.2	Person-Item maps	237
8.3	Logistic versus normal estimates	239
8.4	Two parameter logistic	242
8.5	3 and 4 PL models	243
8.6	Four ordered response alternatives	244
8.7	A multiple choice ability item	246
12.1	The basic congeneric model is one latent (true score) factor accounting for the correlations of multiple observed scores. If there are at least 4 observed variables, the model is identified. For fewer variables, assumptions need to be made (i.e., for two parallel tests, the path coefficients are all equal.)	270
12.2	Six variables with two factors. This notation shows the error of measurement in the observed and latent variables. If $g > 0$, then the two factors are correlated.	278
12.3	A ScatterPlot Matrix, SPLOM, of the six variables.	279

12.4	A scree plot of the eigen values of the simulated data suggests that two factors are the best representation of the data. Compare this to the two correlated factor problem, Figure 12.6, and the three correlated factor problem, Figure 12.9	280
12.5	Six variables loading on 2 correlated factors	288
12.6	Scree plot of two correlated factors. Compare to Figure 12.4	289
12.7	The correlation between two factors may be modeled by a g, general, factor. This representation shows all the errors that need to be estimated.	294
12.8	The correlation between two factors may be modeled by a g, general, factor. This representation is somewhat more compact than the previous figure (12.7.)	296
12.9	Scree plot of three correlated factors. Compare to the two uncorrelated factors, Figure 12.4, and the two correlated factors, ??	297
12.10	The correlation between three factors may be modeled by a g, general, factor.	298
12.11A	hierarchial solution to the three correlated factors problem.	302
12.12A	bifactor solution to the three correlated factors problem.	304
13.1	9 cognitive variables (adapted from Raykov and Marcoulides, 2006)	307
13.2	The Linear Structural Relations (LISREL) model integrates two measurement models with one regression model. How well are the X's represented by the latent variables (factors) Xi, and how well are the Y variables represented by the factors etas.	317
14.1	Determining the number of factors to extract from 24 variables generated with a simple structure. The left hand panel shows the scree plot, the right hand panel a VSS plot. Notice the inflection at two factors, suggesting a two factor solution	340
14.2	Factor loadings for 24 items on two dimensions.	341
14.3	Determining the number of factors to extract from 24 variables generated with a simple structure with 5-point items. The left hand panel shows the scree plot, the right hand panel a VSS plot. Compare with Figure 14.4	346
14.4	24 variables loading on two factors for categorical items. Compare with Figure ??	347
14.5	24 variables, simple structure. Items are constrained to have 3 categories	351
14.6	Determining the number of factors to extract from 24 variables generated with a simple structure for dichotomous items The left hand panel shows the scree plot, the right hand panel a VSS plot. Compare with Figures 14.4 and 14.4.1	354
14.7	24 variables, simple structure. Items are constrained to be dichotomous.	355
14.8	Determining the number of factors to extract from 24 variables generated with a circumplex structure. The left hand panel shows the scree plot, the right hand panel a VSS plot. Notice the inflection at two factors, suggesting a two factor solution	357
14.9	Factor loadings for 24 items on two dimensions. Given that the data were generated to reflect uniform locations around a two dimensional space, the circular ordering of loadings is not surprising.	358
14.10	SPLM of the first 6 variables showing the effect of skew. Note how the correlations of items with opposite skew are very attenuated.	365

14.11	The factor structure of very skewed items recovers the space quite well, at least in terms of angular location. The loadings are less than they should be given the data generation algorithm.	366
14.12	A two dimensional solution does not fit very well.	367
14.13	A two dimensional solution does not fit very well, but a 4 factor model in two space matches the generating function very well.	372
14.14	An alternative solution is to group the variables into “testlets” or “homogeneous item composites” (HICs) and then to examine the structure of the HICs.	377
15.1	The direct and indirect effect of three predictors upon a criterion variable. The “real”, causal variable is missing from the model.	381
15.2	The direct and indirect effect of four predictors upon a criterion variable	383
15.3	The direct effect of three predictors upon a criterion variable	389
15.4	The direct and indirect effect of three predictors upon a criterion variable using sem	394
15.5	Faulty inference can be the result of a misspecified model.	395
15.6	Correlated errors not associated with the criterion.	397
15.7	Fixing the correlated errors paths.	398
15.8	Changing the direction of causation.	400
15.9	Good fit does not imply “causality”—the problem of incorrect inference.	407
15.10	The correct model does not necessarily fit better.	411
17.1	The scree and parallel analyses of the original data matrix suggest that two or perhaps three factors are most appropriate.	420
17.2	The Very Simple Structure criterion suggests 1 factor of complexity 1 or 2 of complexity 2.	420
17.3	A simple structure graph can show just the highest loading for each item	425
A.1	A boxplot with an added stripchart summarizes basic distributional properties of the data.	439
A.2	A scatter plot matrix of the data can be modified to give histograms as well as the correlations.	439
E.1	Scatter plot matrix (SPLOM) of 5 extraversion items for 200 subjects.	467

List of Tables

2.1	The theory of data provides a 3 x 2 x 2 taxonomy for various types of measures	18
2.2	Simulated chess tournament	19
2.3	Three alternative solutions to the chess problem of Table 2.2	20
2.4	A hypothetical response matrix for questions 4-6 about social interaction with an attractive stranger.	24
2.5	Hypothetical results from a speed dating study.	25
2.6	Mohs' scale of mineral hardness.	26
2.7	The Beaufort scale of wind intensity	27
2.8	Paired comparisons of nine vegetables	30
2.9	qnorm function applied to the vegetable data	31
2.10	Models are approximations to the data	31
2.11	Modeled probability of choice based upon the modeled scale values	32
2.12	Residuals = data - model	32
2.13	Basic summary statistics of the residuals suggest that they are very small	32
2.14	Comparing Thurstonian scaling to alternative models.	33
2.15	Airline distances between 11 American cities taken from the cities data set.	35
2.16	Two dimensional representation for 11 American cities.	35
2.17	Midpoint orders provide partial metric information	38
2.18	Midpoint orders imply partial ordering of distances	40
2.19	Bogardus Social Distance Scale	41
2.20	A Guttman Scale	41
3.1	Six observations on seven participants	49
3.2	Four types of scales and their associated statistics	50
3.3	Tukey's five number summary	52
3.4	Sorting the data shows relationships	54
3.5	Finding the median by interpolation	56
3.6	Six estimates of central tendency	59
3.7	Effect of reciprocal transformation upon means	60
3.8	Circular Statistics	62
3.9	Average class size depends upon point of view	63
3.10	Class from the students' point of view	63
3.11	Raw scores and transformed scores	72
3.12	Commonly used distributions	78

3.13	Robust statistics for dirty data	80
3.14	Tukey's ladder of transformations	81
4.1	Galton's height data set	86
4.2	Finding a variance/covariance matrix	89
4.3	Variance, Covariance, Correlation and Residuals	91
4.4	Confidence interval of a correlation	94
4.5	Testing two <i>independent correlations</i>	96
4.6	The difference between two dependent correlations	97
4.7	Testing dependent correlations	97
4.8	Alternatives to the Pearson r	99
4.9	Point bi-serial correlation	101
4.10	The phi, ϕ , coefficient	104
4.11	The basic table for a phi coefficient expressed in proportions	104
4.12	The effect of cut points upon <i>polychoric</i> and <i>phi</i> correlations	107
4.13	Comorbidity and correlation	108
4.14	The basic four fold table	108
4.15	Two by Two tables and the direction of inference	110
4.16	Similarity measures	111
4.17	Base rates and inference	112
4.18	Cohen's kappa	112
4.19	Hypothetical twin data	113
4.20	Intra Class Correlation	115
4.21	Pearson r vs. circular r	116
4.22	Alternative Estimates of <i>effect size</i>	117
4.23	The effect of range restriction on correlation	119
4.24	Spurious correlations of ratios or differences	120
4.25	Descriptive statistics of skew example	123
5.1	An example of suppression	129
5.2	Linear model analysis of interaction data	130
5.3	Using <code>partial.r</code> to find a matrix of partial correlations	133
5.4	logistic regression	136
5.5	Poisson regression	138
5.6	Creating a diurnal data set	139
6.1	Creating a correlation matrix from a factor matrix	144
6.2	Eigenvalue decomposition of a matrix	147
6.3	First principal component of the R matrix	149
6.4	Principal Axis Factor Analysis with one factor	152
6.5	One factor maximum likelihood solution	155
6.6	A two component model	156
6.7	A two factor model	157
6.8	Factors versus components: the effect an additional variable	160
6.9	Principal components and additional variables	161
6.10	Unrotated factor solution to 8 physical variables	164
6.11	Varimax rotation of 8 physical variables	165
6.12	Oblique factor solution	166

6.13	An example hierarchical correlation matrix	169
6.14	Schmid Leiman transformations	171
6.15	Confirmatory factoring of four congeneric measures	179
6.16	sem vs. factanal for four congeneric measures	180
6.17	Testing for tau equivalence	181
6.18	Multidimensional scaling of 24 mental tests	182
6.19	<code>kmeans</code> of two simulated dimensions	186
6.20	ICLUST of a circumplex	189
6.21	Comparing clusters and factors	190
6.22	Correlation between factor scores and factor score estimates	192
7.1	Correcting for attenuation	200
7.2	Parallel, τ equivalent, and congeneric tests	202
7.3	Estimating the parameters of four congeneric tests	202
7.4	Coefficient α may be found using the <code>alpha</code> function in <i>psych</i> . The analysis is done for 5 neuroticism items taken from the <code>bfi</code> data set.	210
7.5	Test variance = general + Group + Error	211
7.6	Using the <code>omega</code> function	216
7.7	Four data sets with equal α reliability estimates	217
7.8	Comparison of 13 estimates of reliability.	220
7.9	Types of reliability	221
7.10	Intraclass Correlation Coefficient	224
7.11	Sources of variances and the Intraclass Correlation Coefficient.	225
8.1	Rasch modeling using <code>ltm</code>	234
8.2	Using the graded response model	245
B.1	Functions used in this chapter. *are part of the <code>psych</code> package.	445