

# Advanced Statistical Procedures

Data = Model + Residual

# The basic data frame

| DV             | IV <sub>1</sub> | IV <sub>2</sub> | SV <sub>1</sub> | SV <sub>2</sub> | CV <sub>1</sub> | CV <sub>2</sub> | ... | CV <sub>N</sub> |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----|-----------------|
| Y <sub>1</sub> | X <sub>11</sub> | X <sub>21</sub> | X <sub>31</sub> | X <sub>41</sub> |                 |                 |     |                 |
| ...            |                 |                 |                 |                 |                 |                 |     |                 |
| Y <sub>n</sub> | X <sub>1n</sub> | X <sub>2n</sub> | X <sub>3n</sub> | X <sub>4n</sub> |                 |                 |     |                 |

# Types of models

- $Y = bX$  ( $X$  is continuous) regression
- $Y = bX$  ( $X$  has two levels) t-test
- $Y = bX$  ( $X$  has  $> 2$  levels) F-test
- $Y = b_1X_1 + b_2X_2$  ( $X$  is continuous) multiple regression
- $Y = b_1X_1 + b_2X_2 + b_3X_1X_2$  ( $X$  is continuous) multiple regression with an interaction term
- $Y = b_1X_1 + b_2X_2 + b_3X_1X_2$  ( $X_i$  categorical) Analysis of Variance

# Analysis of Covariance

- We want to remove the effect of some other predictor
- $Y = b_1X_1 + b_2X_2 + b_3X_1X_2 + Z$
- ( $X_i$  categorical,  $Z$  continuous) Analysis of Covariance

# The general linear model

- `model = lm(y ~ x1 + x2 + x1*x2, data = my.data)`
- But the product term is correlated with `x1` and `x2`, and so we need to 0 center (subtract the mean) from each predictor
  - `cen.data.df <- data.frame(scale (my.data,scale=FALSE))`
  - `model <- lm(y ~ x1 + x2 + x1*x2, data = cen.data.df)`
  - `summary(model)`

# Analysis of Variance

- If  $X_i$  are categorical, we can create a “factor” by
- `X1cat <- as.factor(X1)`
- `X2cat <- as.factor(X2) ...`
- and then
- `model <- aov(y ~ x1cat + x2cat + x1cat*x2cat,  
data = my.data)`
- `summary(model)`
- `print(model.tables(model, “means”),digits=2)`

# Multivariate Analysis

- Suppose we have multiple predictors and we want to understand their structure.
- We can find the sum of all the predictors to get a total score, or we can find the sum of some subset of predictors to get total scores on subsets or factors of the data.
- How many factors are there in the data?

# Factor Analysis and Principal Components

- We try to model the correlations with fewer variables
- $R = FF' + U^2$  (Factor Analysis)
- $R = CC'$  (Components analysis)

# 9 mental tests from Harman-Holzinger

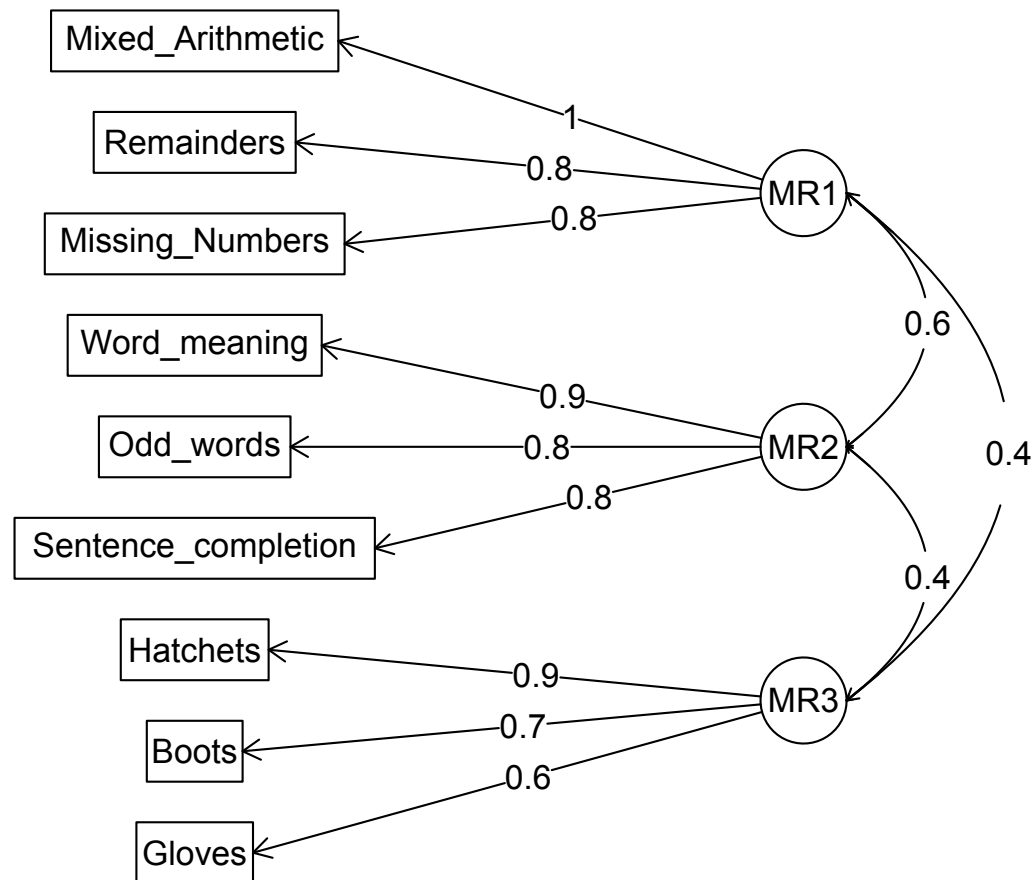
|                     | Harman.Holzinger |      |      |      |      |      |      |      |      |
|---------------------|------------------|------|------|------|------|------|------|------|------|
|                     | V1               | V2   | V3   | V4   | V5   | V6   | V7   | V8   | V9   |
| Word_meaning        | 1.00             | 0.75 | 0.78 | 0.44 | 0.45 | 0.51 | 0.21 | 0.30 | 0.31 |
| Sentence_completion | 0.75             | 1.00 | 0.72 | 0.52 | 0.53 | 0.58 | 0.23 | 0.32 | 0.30 |
| Odd_words           | 0.78             | 0.72 | 1.00 | 0.47 | 0.48 | 0.54 | 0.28 | 0.37 | 0.37 |
| Mixed_Arithmetic    | 0.44             | 0.52 | 0.47 | 1.00 | 0.82 | 0.82 | 0.33 | 0.33 | 0.31 |
| Remainders          | 0.45             | 0.53 | 0.48 | 0.82 | 1.00 | 0.74 | 0.37 | 0.36 | 0.36 |
| Missing_Numbers     | 0.51             | 0.58 | 0.54 | 0.82 | 0.74 | 1.00 | 0.35 | 0.38 | 0.38 |
| Gloves              | 0.21             | 0.23 | 0.28 | 0.33 | 0.37 | 0.35 | 1.00 | 0.45 | 0.52 |
| Boots               | 0.30             | 0.32 | 0.37 | 0.33 | 0.36 | 0.38 | 0.45 | 1.00 | 0.67 |
| Hatchets            | 0.31             | 0.30 | 0.37 | 0.31 | 0.36 | 0.38 | 0.52 | 0.67 | 1.00 |

```
library(psych)  
data(Harman,package="psych")  
Harman.Holzinger
```



# 3 correlated factors

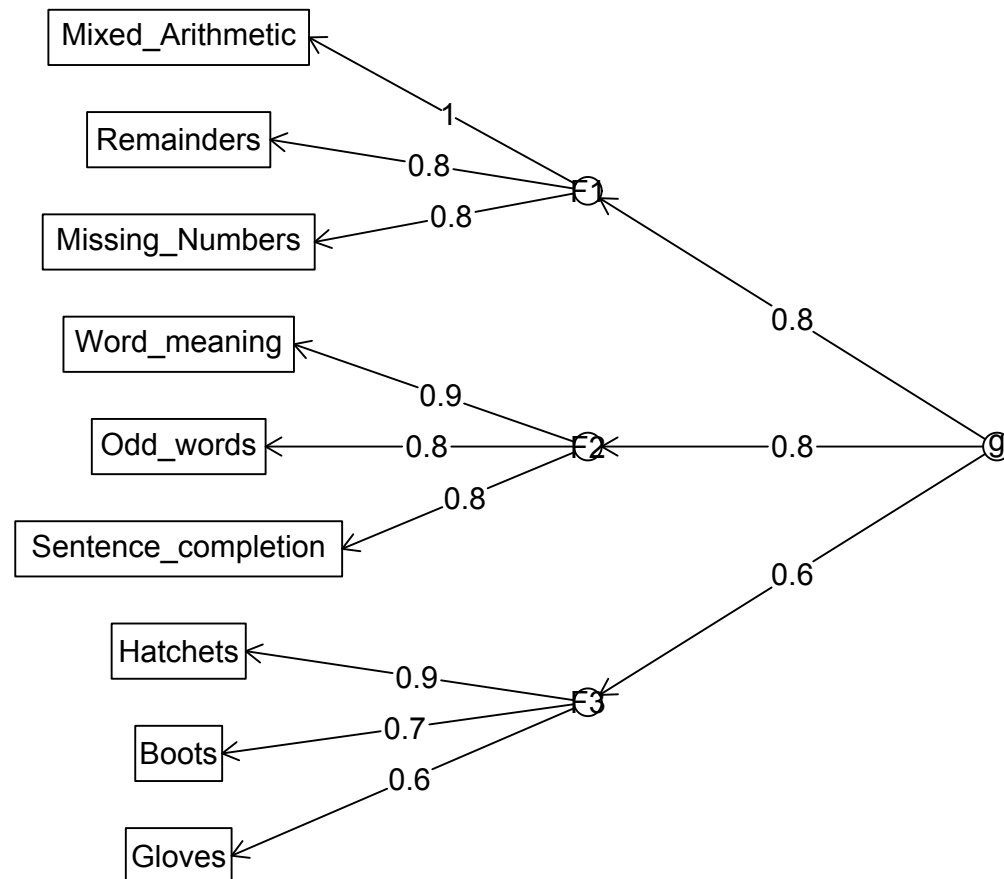
fa.diagram(fa(Harman.Holzinger,3))  
Factor Analysis



# A hierarchical solution

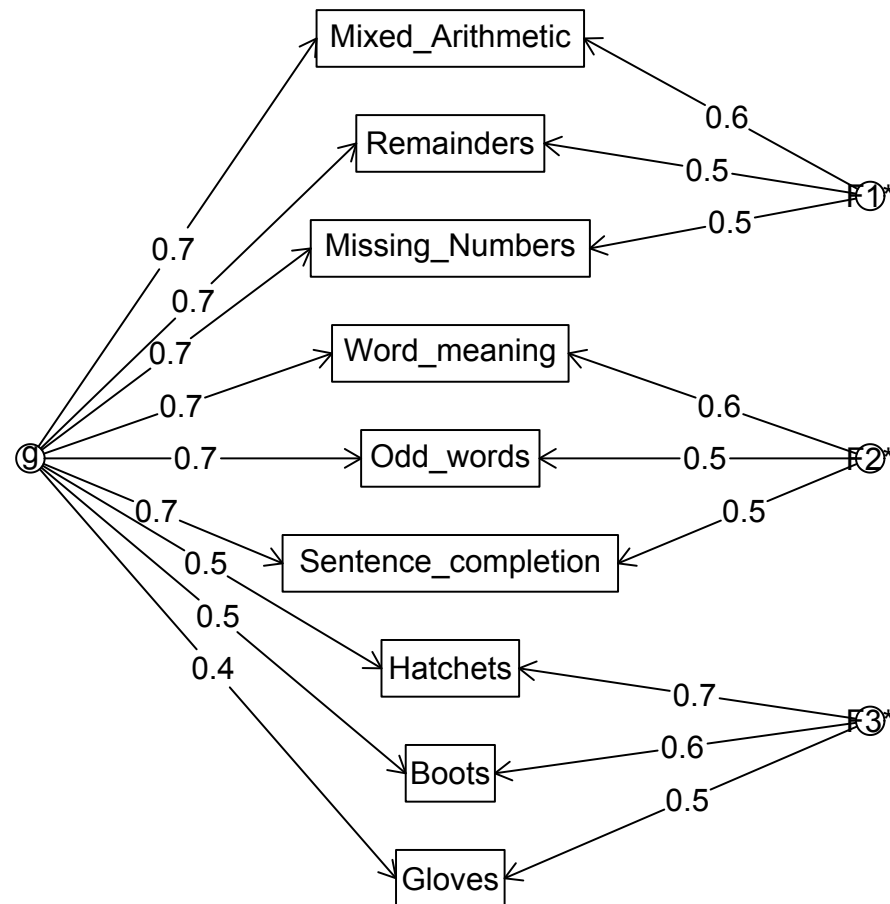
`omega(Harman.Holzinger,sl=FALSE)`

Omega



# A general factor solution

## Holzinger-Harman Problem



# library(psych) data(msq) names(msq)

```
[1] "MSQ_Time"      "active"      "afraid"      "alert"       "alone"       "angry"       "aroused"     "ashamed"     "astonished"  "at-ease"     "at-rest"
[12] "attentive"    "blue"        "bored"       "calm"        "clutched-up" "confident"   "content"     "delighted"   "depressed"   "determined"  "distressed"
[23] "drowsy"       "dull"        "elated"      "energetic"   "enthusiastic" "excited"     "fearful"     "frustrated"  "full-of-pep" "gloomy"      "grouchy"
[34] "guilty"       "happy"       "hostile"     "inspired"    "intense"     "interested"  "irritable"   "jittery"     "kindly"      "lively"      "lonely"
[45] "nervous"      "placid"      "pleased"     "proud"       "quiescent"   "quiet"       "relaxed"     "sad"         "satisfied"   "scared"     "scornful"
[56] "serene"       "sleepy"      "sluggish"    "sociable"    "sorry"       "still"       "strong"      "surprised"   "tense"       "tired"      "unhappy"
[67] "upset"        "vigorous"    "wakeful"     "warmhearted" "wide-awake"  "anxious"     "idle"        "cheerful"    "inactive"    "tranquil"   "EA"
[78] "TA"          "PA"         "NegAff"      "Extraversion" "Neuroticism" "Lie"         "Sociability" "Impulsivity" "MSQ_Round"   "scale"      "ID"
[89] "exper"       "condition"   "TOD"         "TOD24"
```

# Form a new data set

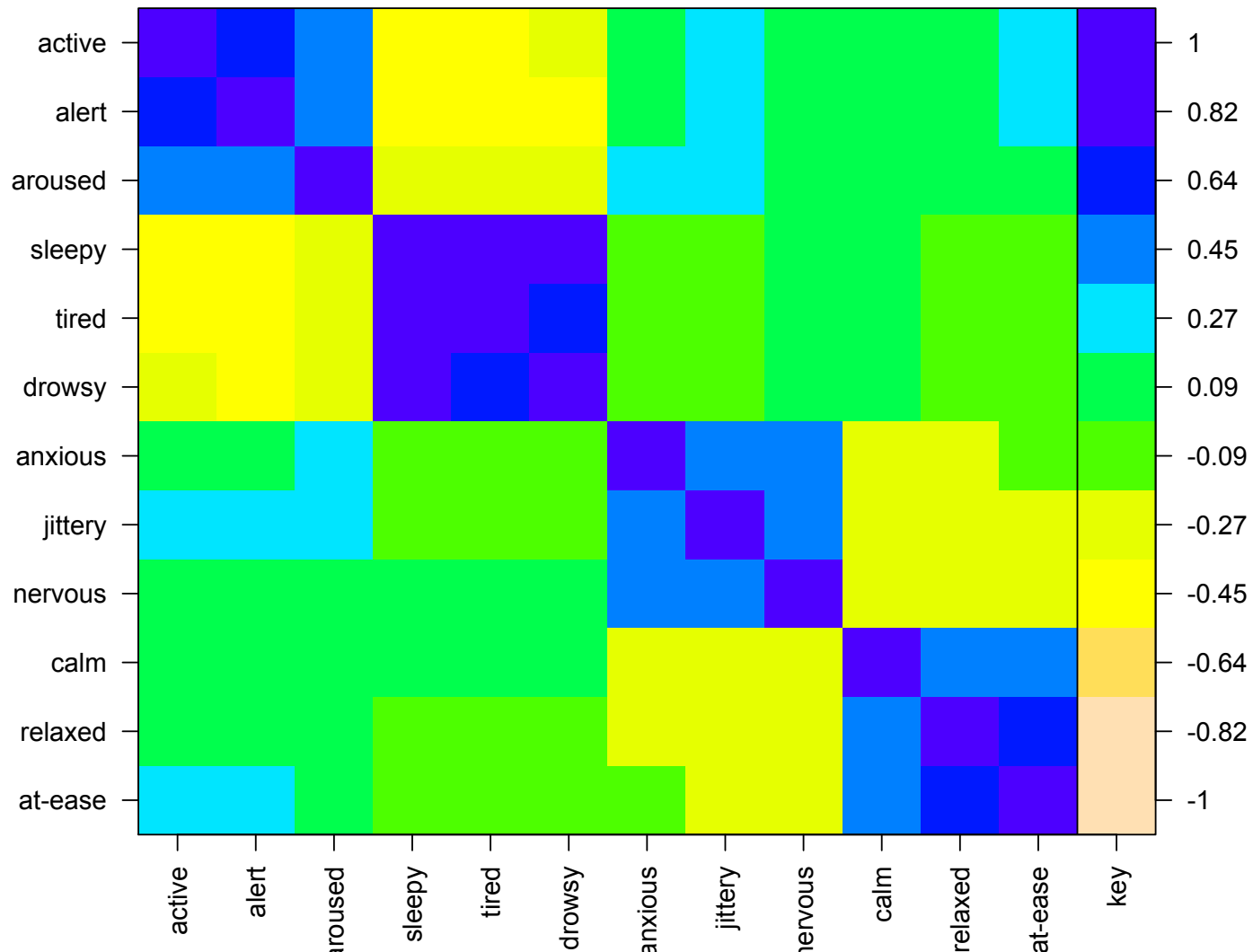
```
> EA.TA <- msq[c  
("active","alert","aroused","sleepy","tired","drowsy","anxious","jittery","nervous","calm","re  
laxed","at-ease")]  
  
> round(cor(EA.TA,use="pairwise"),2)
```

|         | active | alert | aroused | sleepy | tired | drowsy | anxious | jittery | nervous | calm  | relaxed | at-ease |
|---------|--------|-------|---------|--------|-------|--------|---------|---------|---------|-------|---------|---------|
| active  | 1.00   | 0.62  | 0.60    | -0.40  | -0.42 | -0.39  | 0.19    | 0.27    | 0.11    | 0.06  | 0.16    | 0.23    |
| alert   | 0.62   | 1.00  | 0.53    | -0.53  | -0.53 | -0.53  | 0.17    | 0.23    | 0.09    | 0.11  | 0.18    | 0.28    |
| aroused | 0.60   | 0.53  | 1.00    | -0.33  | -0.35 | -0.32  | 0.22    | 0.29    | 0.17    | 0.01  | 0.09    | 0.15    |
| sleepy  | -0.40  | -0.53 | -0.33   | 1.00   | 0.81  | 0.85   | -0.04   | -0.12   | 0.02    | 0.03  | -0.07   | -0.12   |
| tired   | -0.42  | -0.53 | -0.35   | 0.81   | 1.00  | 0.78   | -0.05   | -0.12   | 0.01    | 0.01  | -0.08   | -0.14   |
| drowsy  | -0.39  | -0.53 | -0.32   | 0.85   | 0.78  | 1.00   | -0.03   | -0.11   | 0.02    | 0.05  | -0.07   | -0.10   |
| anxious | 0.19   | 0.17  | 0.22    | -0.04  | -0.05 | -0.03  | 1.00    | 0.45    | 0.51    | -0.25 | -0.24   | -0.19   |
| jittery | 0.27   | 0.23  | 0.29    | -0.12  | -0.12 | -0.11  | 0.45    | 1.00    | 0.47    | -0.28 | -0.30   | -0.22   |
| nervous | 0.11   | 0.09  | 0.17    | 0.02   | 0.01  | 0.02   | 0.51    | 0.47    | 1.00    | -0.27 | -0.28   | -0.27   |
| calm    | 0.06   | 0.11  | 0.01    | 0.03   | 0.01  | 0.05   | -0.25   | -0.28   | -0.27   | 1.00  | 0.54    | 0.58    |
| relaxed | 0.16   | 0.18  | 0.09    | -0.07  | -0.08 | -0.07  | -0.24   | -0.30   | -0.28   | 0.54  | 1.00    | 0.61    |
| at-ease | 0.23   | 0.28  | 0.15    | -0.12  | -0.14 | -0.10  | -0.19   | -0.22   | -0.27   | 0.58  | 0.61    | 1.00    |

# Graphical Displays show structure

```
cor.plot(cor(EA.TA,use="pairwise"),colors=TRUE,zlim=c(-1,1),main="Energetic and Tense")
```

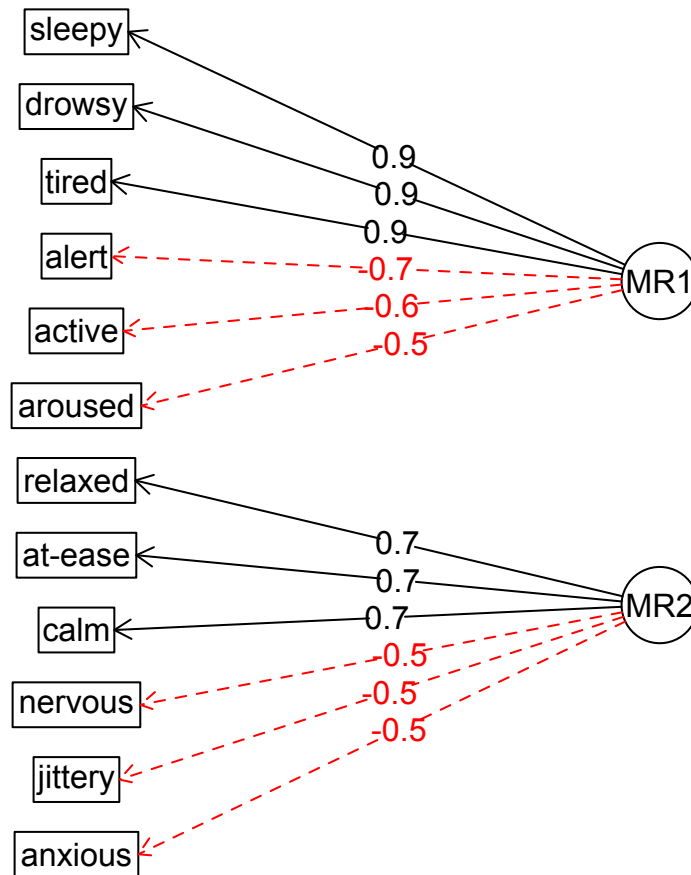
**Energetic and Tense**



# Graphical Display

`fa.diagram(fa(EA.TA,2))`

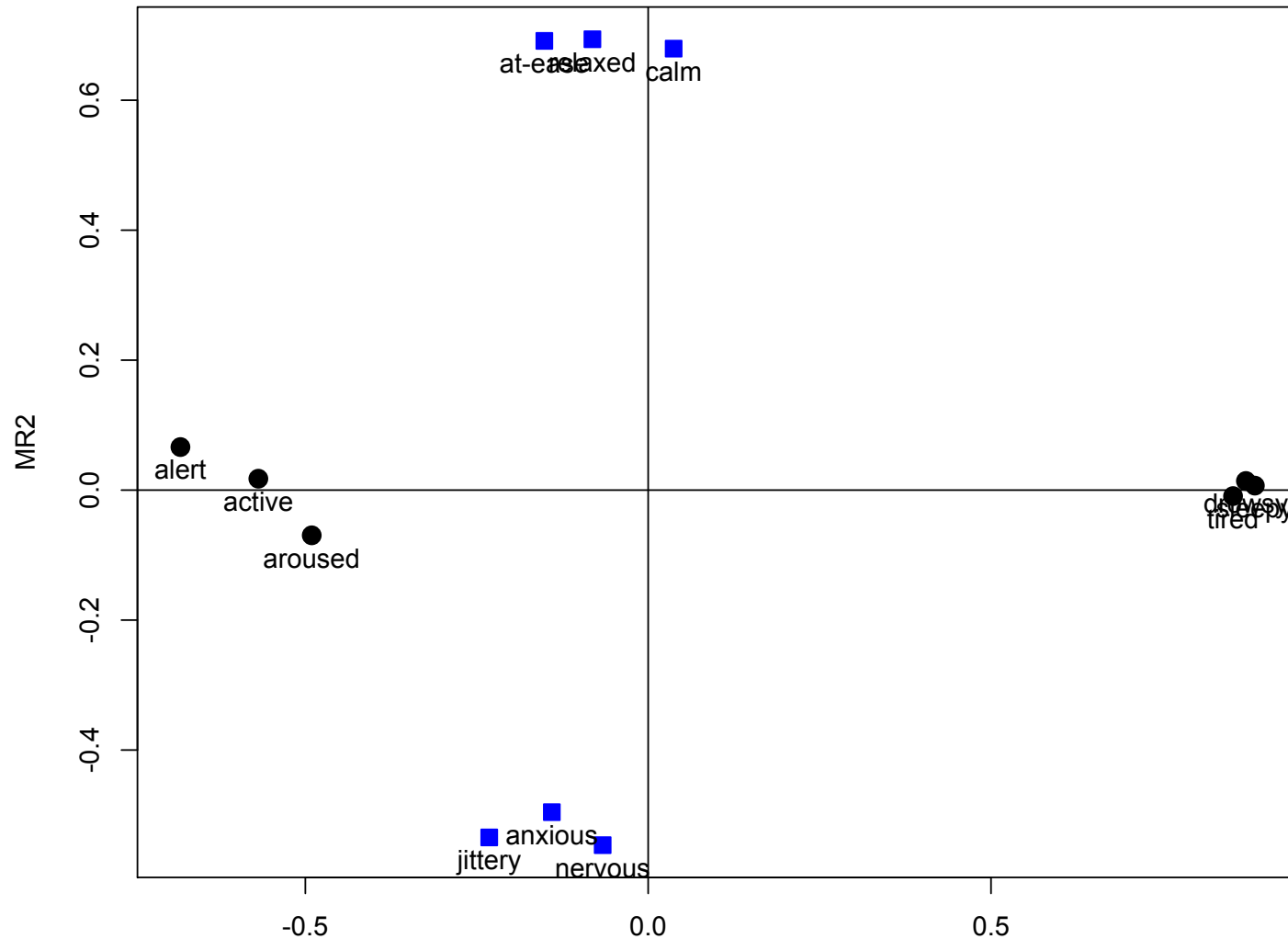
Factor Analysis



# Another way to show it

```
factor.plot(fa(EA.TA,2),labels=colnames(EA.TA))
```

Factor plot



# Score scales by using a keys vector

```
> keys <- make.keys(12,list(EA=c(1:3,-4,-5,-6),TA=c(7:9,-10,-11,-12)))
> keys
      EA TA
[1,]  1  0
[2,]  1  0
[3,]  1  0
[4,] -1  0
[5,] -1  0
[6,] -1  0
[7,]  0  1
[8,]  0  1
[9,]  0  1
[10,] 0 -1
[11,] 0 -1
[12,] 0 -1
> ea.ta.scores <- score.items(keys,EA.TA)
```

# Output of scoring program

```
> ea.ta.scores
Call: score.items(keys = keys, items = EA.TA)
(Unstandardized) Alpha:
      EA    TA
alpha 0.87 0.75
Average item correlation:
      EA    TA
average.r 0.54 0.34
Guttman 6* reliability:
      EA    TA
Lambda.6 0.9 0.77
Scale intercorrelations corrected for
attenuation
  raw correlations below the diagonal, alpha
  on the diagonal
  corrected correlations above the diagonal:
      EA    TA
EA  0.87 -0.02
TA -0.02  0.75
>
```

|         | EA    | TA    |
|---------|-------|-------|
| active  | 0.65  | -0.02 |
| alert   | 0.73  | -0.07 |
| aroused | 0.56  | 0.07  |
| sleepy  | -0.84 | 0.02  |
| tired   | -0.82 | 0.03  |
| drowsy  | -0.83 | 0.01  |
| anxious | 0.06  | 0.36  |
| jittery | 0.25  | 0.53  |
| nervous | 0.06  | 0.55  |
| calm    | 0.01  | -0.66 |
| relaxed | 0.14  | -0.69 |
| at-ease | 0.22  | -0.69 |

Additional output includes scores