Psychology 405: Psychometric Theory
More on Correlations

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Outline

Predicting scores: Question 1

1. If a person has a GRE verbal score of 680, then what would you expect his/her GRE quantitative score to be?
2. Mean GRE V = 600 SD = 80 r = .72
3. Mean GRE Q = 650 SD = 100
4. Observe GRE V = 680
5. Predicted GRE Q = ?

Predicting scores: Answer 1

1. If a person has a GRE verbal score of 680, then what would you expect his/her GRE quantitative score to be?
2. Mean GRE V = 600 SD = 80 r = .72
3. Mean GRE Q = 650 SD = 100
4. Observe GRE V = 680
5. z GRE V = (680- 600)/80 = 1.0
6. predicted z GRE Q = rxy zx = .72 * (1) =.72
7. predicted GRE Q = .72 * 100 + 650 = 722

Predicting scores: Question 2

1. If a person has a GRE Quant score of 722, then what would you expect his/her GRE Verbal score to be?
2. Mean GRE V = 600 SD = 80 r = .72
3. Mean GRE Q = 650 SD = 100
4. Observe GRE Q = 722
5. Predicted GRE V = ?

Predicting scores: Answer 2

1. If a person has a GRE Quant score of 722, then what would you expect his/her GRE Verbal score to be?
2. Mean GRE V = 600 SD = 80 r = .72
3. Mean GRE Q = 650 SD = 100
4. Observe GRE Q = 722
5. Predicted GRE V = ?
6. z GRE Q = (722- 650)/100 = .72
7. predicted z GRE V = rxy zx = .72 * (.72) =.52
8. predicted GRE Q = .52 * 100 + 600 = 652
9. Note that although 680 predicts 722, 722 predicts 652.

Predicting Scores: Question 3

1. For a person with an anxiety score of 16, what is the expected GPA?
2. Anxiety Mean = 12 sd = 4 r = -.39
3. GPA Mean = 3.0 sd = .5

Predicting Scores: Answer 3

1. For a person with an anxiety score of 16, what is the expected GPA?
2. Anxiety Mean = 12 sd = 4 r = -.39
3. GPA Mean = 3.0 sd = .5
4. z anx = (16- 12)/4 = 1.0
5. predicted z gpa = rxy zx = -.3.9 * (1) =-.39
6. predicted gpa = -.39 * .5 + 3 = 2.805

Partial Correlations: Question

1. Assuming the classical model of partial correlations, what is the correlation between GRE Quantitative and GPA with GRE Verbal held constant?
2. $r_{GREQ,GPA} = .34$
3. $r_{GREQ,V} = .72$ $r_{GREV,GPA} = .38$
4. We want to find the covariance of Q and GPA without V.
5. All correlations are $= \frac{C_{xy}}{\sqrt{V_x V_y}}$. So we just need to find the Covariances and Variances.

Partial Correlations: Answer

1. Assuming the classical model of partial correlations, what is the correlation between GRE Quantitative and GPA with GRE Verbal held constant?
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3. $r_{GREQ,V} = .72$ $r_{GREV,GPA} = .38$
4. All correlations are $= \frac{C_{xy}}{\sqrt{V_x V_y}}$. So we just need to find the Covariances and Variances.
5. partial $r_{xy.z} = \frac{r_{xy} - r_{xz} * r_{yx}}{\sqrt{(1 - r_{xz}^2) * (1 - r_{yx}^2)}}$
6. $r_{qgpa.v} = \frac{(.34 - .72 * .38)}{\sqrt{(.482 * .856)}} = .103$ partial
7. part r $= \frac{r_{xy} - r_{xz} * r_{yx}}{\sqrt{1 - r_{xz}^2}} = .096$ part

Multiple Correlation: Question

1. What is the multiple correlation of GRE V and GRE Q with rated quality of the MA?
2. r GRE V, MA = .32 r GRE Q, MA = .29 r GRE V,Q =.72
3. $beta_{y.x} = \frac{r_{xy} - r_{xz} * r_{yz}}{1 - r_{xz}^2}$
4. $R^2 = \Sigma \beta_i r_{x_i y}$

Multiple Correlation: Answer

1. What is the multiple correlation of GRE V and GRE Q with rated quality of the MA?
2. r GRE V, MA = .32 r GRE Q, MA = .29 r GRE V,Q =.72
3. $beta_{y.x} = \frac{r_{xy} - r_{xz} * r_{yz}}{1 - r_{xz}^2}$
4. beta GRE V, MA= (.32- .72*.29)/(1-.722) = .231
5. beta GRE Q, MA = (.29- .72*.32)/(1-.722) = .124
6. $R^2 = \beta_{y.x} * r_{xy} + \beta_{y.z} * r_{yz} ...$
7. R2 = beta GRE Q, MA * r GRE Q, MA + beta GRE V, MA* r GRE V, MA =
8. R2 = .124 * .29 + .231 * .32 = .108
9. R = .329

Unit Weighted Multiple R

1. What is the unit weighted correlation of GREV and GRE Q with MA?
2.

Variable	GREV	GREQ	MA
GREV	1.00	0.72	0.32
GREQ	0.72	1.00	0.29
MA	0.32	0.29	1.00

3. All correlations are $= \frac{C_{xy}}{\sqrt{V_x V_y}}$. So we just need to find the Covariances and Variances.

Unit Weighted Multiple R

1. Weight the two predictors equally
2.

Variable	GREV	GREQ	MA
GREV	1.00	0.72	0.32
GREQ	0.72	1.00	0.29
MA	0.32	0.29	1.00

3. All correlations are $= \frac{C_{xy}}{\sqrt{V_x V_y}}$. So we just need to find the Covariances and Variances.
4. $C_{v+q,MA} = .32 + .29 = .61$
5. $V_{v+q} = 1.00 + .72 + .72 + 1.00 = 3.44$
6. $r_{v+q,MA} = \frac{.61}{\sqrt{3.44*1}} = .329$

Correlating two composites – unit weights

Table: Ability and Performance

Hypothetical relationships

Variable	GREV	GREQ	GREA	GPA	Pre	MA
GREV	1.00	0.72	0.54	0.38	0.32	0.27
GREQ	0.72	1.00	0.48	0.34	0.29	0.24
GREA	0.54	0.48	1.00	0.55	0.47	0.39
GPA	0.38	0.34	0.55	1.00	0.42	0.35
Pre	0.32	0.29	0.47	0.42	1.00	0.30
MA	0.27	0.24	0.39	0.35	0.30	1.00

1. What is the unit weighted correlation between the ability measures and the performance measures?
2. All correlations are $= \frac{C_{xy}}{\sqrt{V_x V_y}}$. So we just need to find the Covariances and Variances.

Correlating two composites – unit weights

Table: Ability and Performance

Hypothetical relationships

Variable	GREV	GREQ	GREA	GPA	Pre	MA
GREV	1.00	0.72	0.54	0.38	0.32	0.27
GREQ	0.72	1.00	0.48	0.34	0.29	0.24
GREA	0.54	0.48	1.00	0.55	0.47	0.39
GPA	0.38	0.34	0.55	1.00	0.42	0.35
Pre	0.32	0.29	0.47	0.42	1.00	0.30
MA	0.27	0.24	0.39	0.35	0.30	1.00

1. $V_{ability} = 1.0 + .72 + .54 + .72 + 1.0 + .48 + .54 + .48 + 1 = 3 * 1 + 2 * (.72 + .54 + .48) = 6.48$
2. $V_{performance} = 3 * 1 + 2 * (.42 + .35 + .30) = 5.14$
3. $C_{ability,performance} = .38 + .34 + .55 + .32 + .29 + .47 + .27 + .24 + .39 = 3.25$
4. $R_{ability,performance} = \frac{3.25}{\sqrt{6.48 * 5.14}} = .563$

Cohen's Set correlation versus unit weighted correlations

1. What is the relationship between two sets of variables? Two alternative answers.
2. Set correlation: 1 - the ratio of determinants
   - $R = 1 - \frac{||\mathbf{R}_{xy}||}{||\mathbf{R}_x||||\mathbf{R}_y||}$
3. Unit weighted correlation
   - $R_{uw} = \frac{\mathbf{1R}_{yx}\mathbf{1}'}{(\mathbf{1R}_{yy}\mathbf{1}')^{.5}(\mathbf{1R}_{xx}\mathbf{1}')^{.5}}$
4. Both are found using the `set.cor` function

set.cor to find multiple correlations from correlation matrices

```
> m1
>  set.cor(4:6,1:3,m1)


     GREV GREQ GREA  GPA  Pre   MA
GREV 1.00 0.72 0.54 0.38 0.32 0.27
GREQ 0.72 1.00 0.48 0.34 0.29 0.24
GREA 0.54 0.48 1.00 0.55 0.47 0.39
GPA  0.38 0.34 0.55 1.00 0.42 0.35
Pre  0.32 0.29 0.47 0.42 1.00 0.30
MA   0.27 0.24 0.39 0.35 0.30 1.00
Call: set.cor(y = 4:6, x = 1:3, data = m1)

Multiple Regression from matrix input

Beta weights
      GPA  Pre   MA
GREV 0.09 0.06 0.06
GREQ 0.05 0.05 0.03
GREA 0.48 0.41 0.34

Multiple R
 GPA  Pre   MA
0.56 0.48 0.40
multiple R2
 GPA  Pre   MA
0.31 0.23 0.16
```

```
Unweighted multiple R
 GPA  Pre   MA
0.50 0.42 0.35
 Unweighted multiple R2
 GPA  Pre   MA
0.25 0.18 0.13

Various estimates of between set correlations
Squared Canonical Correlations
[1] 4.1e-01 6.0e-05 1.6e-06

 Average squared canonical correlation =  0.14
 Cohen's Set Correlation R2 =  0.41
Unweighted correlation between the two sets =  0.56
```

Just use Verbal and Quant

```
> m1
> set.cor(4:6,1:2,m1)
```

```
     GREV GREQ GREA  GPA  Pre   MA
GREV 1.00 0.72 0.54 0.38 0.32 0.27
GREQ 0.72 1.00 0.48 0.34 0.29 0.24
GREA 0.54 0.48 1.00 0.55 0.47 0.39
GPA  0.38 0.34 0.55 1.00 0.42 0.35
Pre  0.32 0.29 0.47 0.42 1.00 0.30
MA   0.27 0.24 0.39 0.35 0.30 1.00

Multiple Regression from matrix input

Beta weights
      GPA  Pre   MA
GREV 0.28 0.23 0.20
GREQ 0.14 0.12 0.09

Multiple R
 GPA  Pre   MA
0.39 0.33 0.28
multiple R2
  GPA   Pre    MA
0.154 0.110 0.077
```

```
 Unweighted multiple R
 GPA  Pre   MA
0.39 0.33 0.27
 Unweighted multiple R2
 GPA  Pre   MA
0.15 0.11 0.08

Various estimates of between set correlations
Squared Canonical Correlations
[1] 2.0e-01 4.2e-05

 Average squared canonical correlation =  0.1
 Cohen's Set Correlation R2 =  0.2
Unweighted correlation between the two sets =  0.44
```

Comorbidity

1. Symptoms are said to be comorbid if one has both symptoms.
   - This is really just one cell in a 2 x 2 table
   - We need base rates as well
2. Consider Anxiety and Depression
   - 50 % of anxiety patients are also depressed
   - 67% of depressed patients are also anxious
   - base rates are 20% for anxiety, 15% for depression

```
comorbidity(.2,.15,.1,c("Anxiety","Depression"))

Call: comorbidity(d1 = 0.2, d2 = 0.15, com = 0.1, labels = c("Anxiety",
    "Depression"))
Comorbidity table
            Anxiety -Anxiety
Depression      0.1     0.05
-Depression     0.1     0.75

implies phi =  0.49  with Yule =  0.87  and tetrachoric correlation of  0.75
and normal thresholds of  1.04 0.84
```