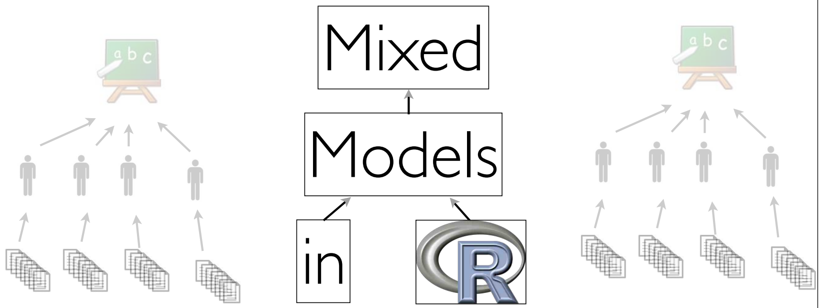
LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN



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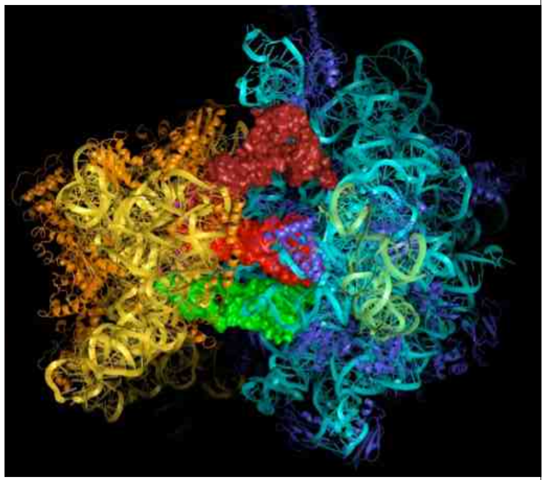
www.nicebread.de

All models are wrong, but some models are useful.

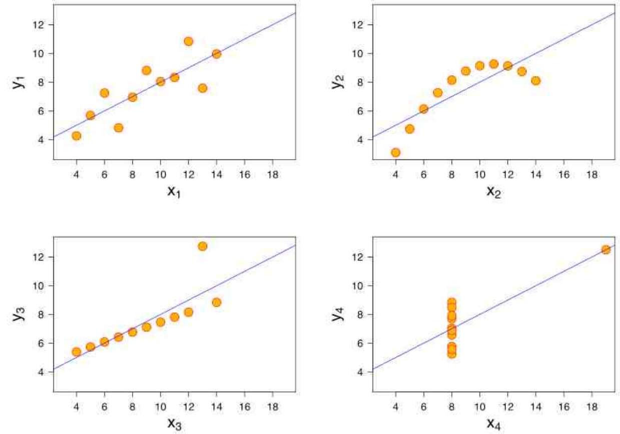
George Box



„But what is the point of fitting a model, if not to be able to understand it?"

Andrew Gelman (2005), p. 460

The beauty/meaningfulness of plotting



Same mean, variance, correlation, and regression in each panel!

Overview

- Basic regression in R
- Extension to MLM: Two-level models
  - Modeling
  - Plotting
  - Understanding
  - Demo data set: Children nested in schools (HSB data set)
- Centering
  - Demo data set: BEER!
- Advanced modeling issues
  - *p*-values, SE for random variance, R^2, etc.
- Three-level longitudinal models
- GLMM: Logistic regression
- Dyadic Diary Data: DDD

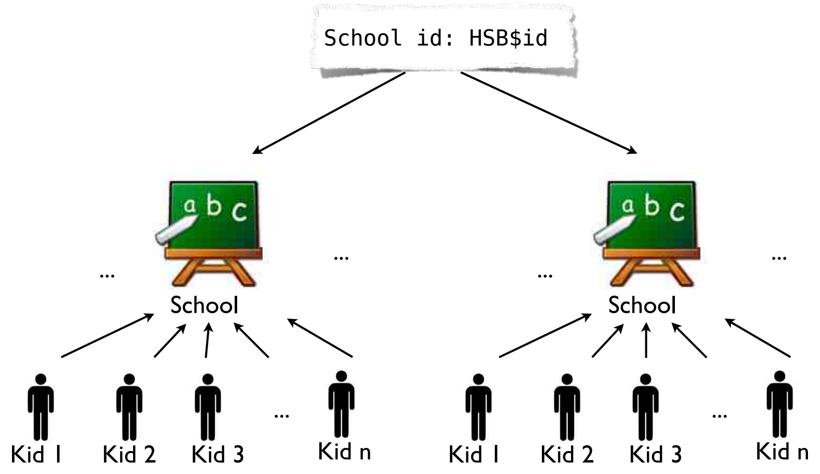
Basic regression

DEMO!

The HSB school data set*

* aka. „the most frequently analyzed data set of the world"
http://nces.ed.gov/surveys/hsb/

- 2 levels: Pupils nested in schools
  - HSB1: data set for level 1, each row is one pupil ➝ pupil variables
  - HSB2: data set for level 2, each row is one school ➝ school variables
- Variables Level 1:
  - minority: Does the pupil belong to a minority? 0 = no, 1 = yes
  - female: 0 = no, 1 = yes
  - ses: Socio-economic status (continuous, grand-mean centered)
  - mathach: mathematical achievement (points in a test)
- Variables Level 2:
  - id: school id
  - sector: 0 = public, 1 = catholic



Data format for lme4: Long format

- Each row is one Level-1 observation
- Level-2 variables are repeated for each Level-1 observation

L2 ID	L1 VARIABLES				L2 VARIABLE
id	minority	female	ses	mathach	sector
...					
2458	1	1	0.022	10.621	Catholic
2458	0	1	0.052	10.138	Catholic
2458	1	0	-0.888	11.333	Catholic
2458	0	1	-0.828	10.352	Catholic
2458	0	1	0.382	19.322	Catholic
2458	1	1	0.272	6.451	Catholic
2458	1	0	-0.728	16.445	Catholic
2458	1	0	-0.048	21.186	Catholic
2458	0	1	0.312	12.293	Catholic
2458	1	1	-0.918	11.960	Catholic
2458	0	1	0.432	9.327	Catholic
2467	0	1	-1.388	7.278	Public
2467	0	0	-0.848	10.811	Public
2467	0	1	-0.308	10.838	Public
2467	1	0	-0.358	15.614	Public

A L2 UNIT

ANOTHER L2 UNIT

Data format for lme4: Long format

- If files are separate for L1 and L2 (as necessary for the HLM software): merge them to one data frame!

```
HSB1 <- read.csv("data/HSB1.csv", dec=",", sep=";")
HSB2 <- read.csv("data/HSB2.csv", dec=",", sep=";")
HSB  <- merge(HSB1, HSB2, by="id")

# define the factors
HSB$sector <- factor(HSB$sector, labels=c("Public", "Catholic"))
HSB$gender <- factor(HSB$female, labels=c("male", "female"))
```


Multiple linear regression in R

$Y = \beta_0 + \beta_1 * X_1 + \beta_2 * X_2 + \ldots + \beta_i * X_i + e$

For example:

$matchach = \beta_0 + \beta_1 * ses + \beta_2 * gender + \beta_3 * minority + e$

```
# Syntax for lm() - both are equivalent:

lm(mathach ~ ses + gender + minority, data = ...)
lm(mathach ~ 1 + ses + gender + minority, data = ...)
```

„1 +" DEFINES THE INTERCEPT β_0. INCLUDED PER DEFAULT, SO IT DOESN'T NEED TO BE SPECIFIED

```
head(HSB) #shows the first six rows
res.lm <- lm(mathach ~ ses + gender + minority, data=HSB)
```

```
> summary(res.lm)

Call:
lm(formula = mathach ~ ses + gender + minority, data = HSB)

Residuals:
     Min       1Q   Median       3Q      Max
-19.6401  -4.5587   0.2658   4.9098  17.5411

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  14.25389    0.11775  121.05   <2e-16 ***
ses           2.68299    0.09873   27.18   <2e-16 ***
genderfemale -1.37665    0.14835   -9.28   <2e-16 ***
minority     -2.83651    0.17198  -16.49   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.263 on 7181 degrees of freedom
Multiple R-squared:  0.1713, Adjusted R-squared:  0.171
F-statistic: 494.8 on 3 and 7181 DF,  p-value: < 2.2e-16
```

estimates of β_i

estimates of p-values

<2e-16
→ $< 2*10^{-16}$
→ 0.0000000000...


```
res.IA <- lm(mathach ~ ses*gender, data=HSB)
res.IA <- lm(mathach ~ ses + gender + sex:gender, data=HSB)
```

```
Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)       13.4749     0.1099 122.657   <2e-16 ***
ses                3.0112     0.1413  21.317   <2e-16 ***
genderfemale      -1.3667     0.1511  -9.043   <2e-16 ***
ses:genderfemale   0.2136     0.1940   1.101    0.271
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.38 on 7181 degrees of freedom
Multiple R-squared:  0.1401, Adjusted R-squared:  0.1397
F-statistic: 389.9 on 3 and 7181 DF,  p-value: < 2.2e-16
```

Moderation of the slop

mathach = $\beta_0 + \beta_1$*ses + β_2*gender + β_3*ses*gender

Re-arrange equation:

mathach = $\beta_0 + \beta_2$*gender + ($\beta_1 + \beta_3$*gender)*ses

mathach = 13.47 - 1.37 * {0=male, 1=female} +
(3.01 + 0.21 * {0=male, 1=female})*ses

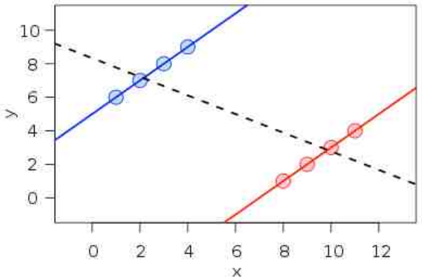
Now we go multilevel / mixed ...

Labels

... not completely exchangeable

- MLM = Multilevel Model
- HLM = Hierarchical Linear Model
- LMM = Linear Multilevel Model
- Random effects models
- Random coefficient models
- Mixed effects models

Simpson's paradox/ ecological fallacy is a paradox in which a trend that appears in different groups of data disappears when these groups are combined, and the reverse trend appears for the aggregate data.



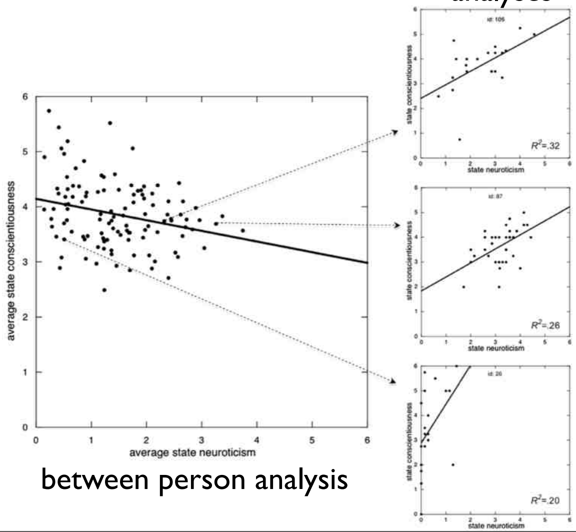
http://en.wikipedia.org/wiki/Simpson%27s_paradox
http://en.wikipedia.org/wiki/Ecological_fallacy

Beckmann, N., Wood, R. E., & Minbashian, A. (2010). **It depends how you look at it: On the relationship between neuroticism and conscientiousness at the within- and the between-person levels of analysis.** *Journal of Research in Personality, 44*, 593–601.

As expected, on the **within-person level** neuroticism **positively** predicted conscientiousness [r = .11], while on the **between-person level** neuroticism **negatively** predicted conscientiousness [r = -.26 to -.45].

Beckmann, Wood, & Minbashian (2010)

exemplary within-person analyses



between person analysis

Further examples for ecological fallacies ...

- Speed-accuracy-trade off
  - Higher speed goes along with **higher** accuracy (between)
  - Higher speed goes along with **lower** accuracy (within)
- Exercise and fatigue
  - More exercise leads to **less** fatigue (between)
  - More exercise leads to **more** fatigue (within)

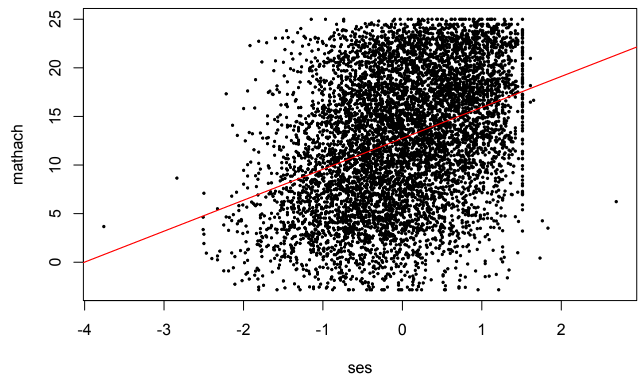
MacKay, D. G. (1982). The problems of flexibility, fluency, and speed–accuracy trade-off in skilled behavior. *Psychol. Rev. 89*, 483–506.
Puetz, T. W., O'Connor, P. J., & Dishman, R. K. (2006). Effects of chronic exercise on feelings of energy and fatigue: A quantitative synthesis. Psychological Bulletin, 132(6), 866–876. doi:10.1037/0033-2909.132.6.866
For a summary, see Kievit, R. A., & Epskamp, S. (2013). Simpson's paradox in psychological science: a practical guide, 1–14. doi:10.3389/fpsyg.2013.00513/abstract

Naïve approaches I: Complete pooling (neglect multilevel structure)

Naïve 1: Complete pooling

```
lm(mathach ~ ses, HSB)
```

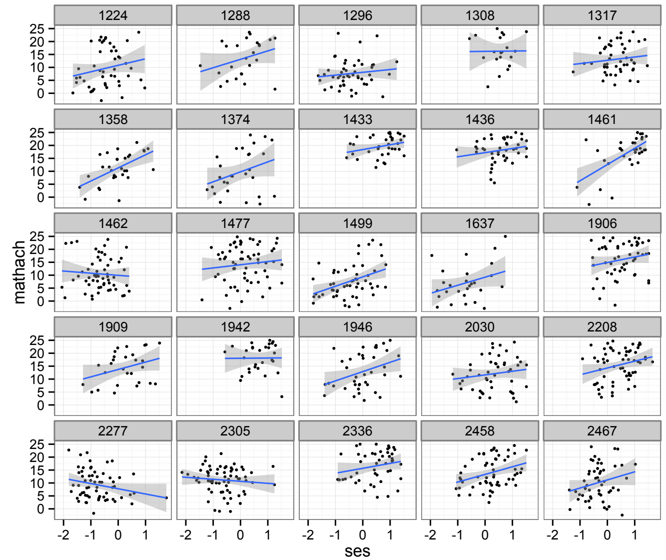
```
b0 = 12.75*** [12.60; 12.90]
b1 =  3.18*** [2.99; 3.37]
```

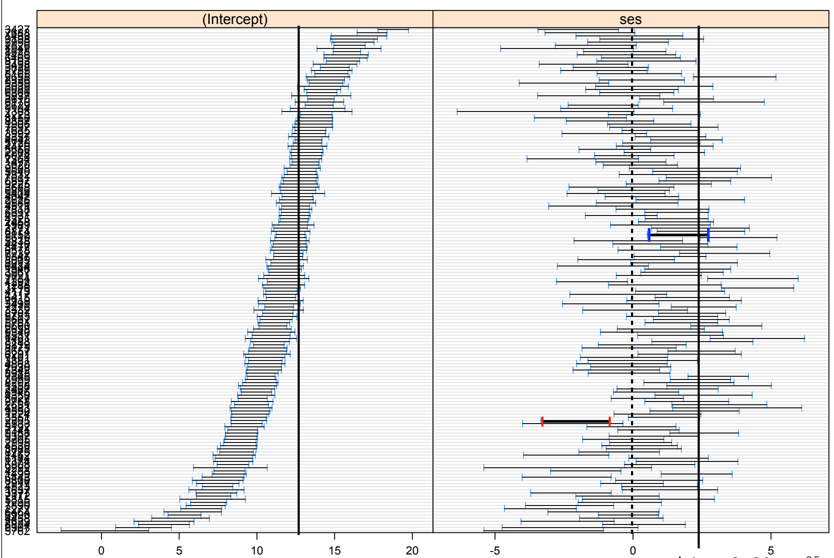


Naïve approaches II: Each unit gets its own regression

Rogosa, D., & Saner, H. (1995). Longitudinal data analysis examples with random coefficient models. Journal of Educational and Behavioral Statistics, 20(2), 149.

Plot for 25 of 160 schools





mean b0 = 12.60 mean b1 = 2.20

Naïve 1: b0 = 12.75*** [12.60; 12.90] b1 = 3.18*** [2.99; 3.37]

Let‘s go MLM ...

One predictor on each level:

Level 1: $Y = \beta_0 + \beta_1 * X_1 + e$ $Y = \beta_0 + \beta_1 * X_1 + e$

Level 2: $\beta_0 = \gamma_{00} + \gamma_{01} * Z_1 + u_0$

$\beta_1 = \gamma_{10} + \gamma_{11} * Z_1 + u_1$

fixed part random part

Mixed equation:

$Y = \boxed{\gamma_{00} + \gamma_{01}*Z_1 + \gamma_{10}*X_1 + \gamma_{11}*Z_1*X_1} + \boxed{u_0 + u_1 * X_1 + e}$

```
# lmer syntax for fixed part:
lmer(Y ~ Z1 + X1 + Z1:X1 + ...,...)
lmer(Y ~ Z1 * X1 + ...,...)

#lmer syntax for random part:
lmer(Y ~ ... + (1|L2Unit),...)      # random intercept (I)
lmer(Y ~ ... + (1 + X1|L2Unit),...) # random I + random slope
lmer(Y ~ ... + (0 + X1|L2Unit),...) # random slope (fixed I)
```

Z * X INCLUDES THE MAIN EFFECTS AS WELL AS THE INTERACTION AND IS SHORT FOR Z + X + Z:X

THE INTERCEPT γ_{00} AND RESIDUAL e IS INCLUDED PER DEFAULT AND NEEDS NO EXPLICIT SPECIFICATION.

Mixed linear models (general for two levels):

X_i = Level I predictor

Level 1: $Y = \beta_0 + \beta_1 * \boxed{X_1} + \beta_2 * X_2 + ... + \beta_i * X_i + e$

random variation of group (i.e. level 2 units) specific intercepts/slopes

Z_i = Level 2 predictor

Level 2: $\beta_0 = \gamma_{00} + \gamma_{01} * \boxed{Z_1} + \gamma_{02} * Z_2 + ... + \gamma_{0j} * Z_j + \boxed{u_0}$

$\beta_1 = \gamma_{10} + \gamma_{11} * \boxed{Z_1} + \boxed{u_1}$

$\beta_2 = \gamma_{20} + \gamma_{21} * \boxed{Z_1} + \gamma_{22} * Z_2 + ... + \gamma_{2j} * Z_j + \boxed{u_2}$

$\boxed{\beta_3 = \gamma_{30} + u_3}$ ← unconditional random coefficient

$\boxed{\beta_4 = \gamma_{40}}$ ← fixed coefficient

1. **The Null-Model: Random effects ANOVA**

2. L1 predictor with random intercepts

3. L1 predictor with random intercepts and slopes

4. Adding a L2 predictor: Cross-level interactions


```
library(lme4)
lmer(mathach ~ 1 + (1|id), HSB)
```

Person level (Level 1):

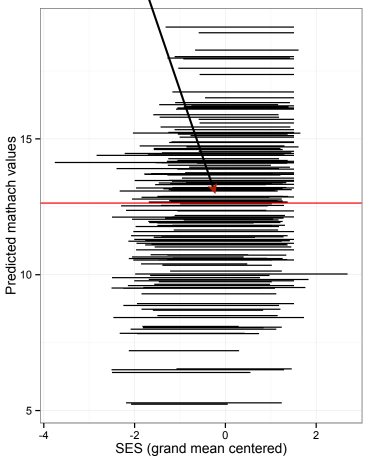
$mathach_{ij} = \beta_{0j} + e_{ij}$

School level (Level 2):

$\beta_{0j} = \gamma_{00} + u_{0j}$

THIS IS A RANDOM EFFECTS ANOVA, FORMULATED AS A MLM MODEL ...

b0 = 12.63 [12.16; 13.12]




```
mathach ~ 1 + (1 | id)

Random effects:
 Groups   Name        Variance Std.Dev.
 id       (Intercept)  8.614   2.935
 Residual             39.148   6.257
Number of obs: 7185, groups: id, 160

Fixed effects:
            Estimate Std. Error t value
(Intercept)  12.6370     0.2444   51.71
```

ICC =
8.614 / (**8.614** + 39.148) =
18 %

1. The Null-Model: Random effects ANOVA

2. **L1 predictor with random intercepts**

3. L1 predictor with random intercepts and slopes

4. Adding a L2 predictor: Cross-level interactions

```
lmer(mathach ~ ses + (1|id), HSB)
```

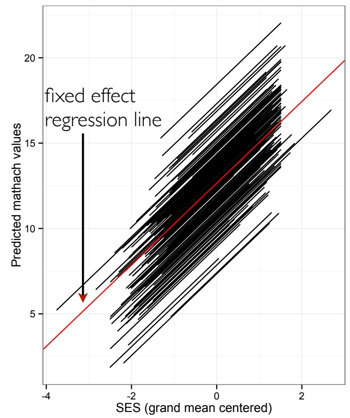
Person level (Level 1):

$mathach_{ij} = \beta_{0j} + \beta_{1j} * ses_{ij} + e_{ij}$

School level (Level 2):

$\beta_{0j} = \gamma_{00} + u_{0j}$
$\beta_{1j} = \gamma_{10}$

b0 = 12.66 [12.27; 13.05]
b1 = 2.39 [2.17; 2.61]




```
mathach ~ ses + (1 | id)

Random effects:
 Groups   Name        Variance Std.Dev.
 id       (Intercept)  4.768   2.184
 Residual             37.034   6.086
Number of obs: 7185, groups: id, 160

Fixed effects:
            Estimate Std. Error t value
(Intercept)  12.6575     0.1880   67.33
ses           2.3902     0.1057   22.61
```


1. The Null-Model: Random effects ANOVA
2. L1 predictor with random intercepts
3. **L1 predictor with random intercepts and slopes**
4. Adding a L2 predictor: Cross-level interactions


```
lmer(mathach ~ ses + (ses|id), HSB)
```

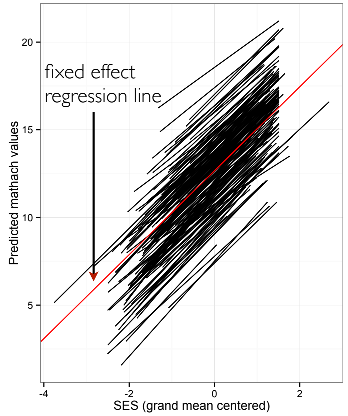
Person level (Level 1):

$mathach_{ij} = \beta_{0j} + \beta_{1j} * ses_{ij} + e_{ij}$

School level (Level 2):

$\beta_{0j} = \gamma_{00} + u_{0j}$
$\beta_{1j} = \gamma_{10} + u_{1j}$

```
b0 = 12.66 [12.32; 13.05]
b1 =  2.39 [2.19; 2.64]
```



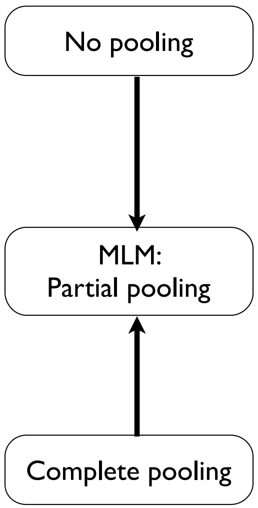

```
mathach ~ ses + (ses | id)

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept)  4.8285  2.1974
          ses          0.4129  0.6426   -0.109
 Residual             36.8302  6.0688
Number of obs: 7185, groups: id, 160

Fixed effects:
            Estimate Std. Error t value
(Intercept)  12.6650     0.1898   66.71
ses           2.3938     0.1181   20.27
```


Fixed effects: Comparison

	b0 (Intercept)	b1 (ses)
Naïve 1: Complete pooling	12.75 [12.60; 12.90]	3.18 [2.99; 3.37]
Naïve 2: Mean of individual regressions	12.60 [12.17; 13.03]	2.20 [1.95; 2.45]
MLM: random intercepts	12.66 [12.27; 13.05]*	2.39 [2.17; 2.61]*
MLM: random intercepts, random slopes	12.66 [12.32; 13.05]*	2.39 [2.19; 2.64]*

* bootstrapped CIs (not symmetric)

Levels of pooling



Model each group separately

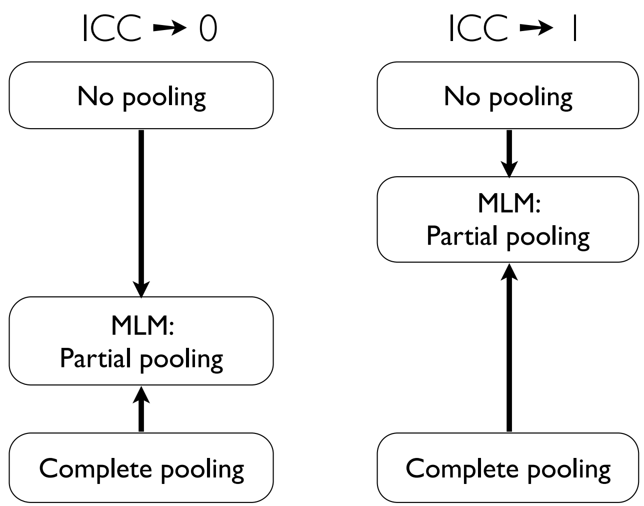
Jointly estimate the overall and the separate model, optimal weighting of L2-units

➞ units with small samples (unreliable estimate) get more shrunken towards the „complete pooling solution"
➞ units with large samples (reliable estimate) are close to the „no pooling" estimate

Neglect clustered structure

Gelman & Hill (2007)

Levels of pooling



Gelman & Hill (2007)

Howto: Extract the estimates of the model

Level-1-Eq: $Y_{ij} = \beta_{0j} + \beta_{1j} X_{ij} + e_{ij}$

Level-2-Eq: $\beta_{0j} = \gamma_{00} + u_{0j}$

$\beta_{1j} = \gamma_{10} + u_{1j}$

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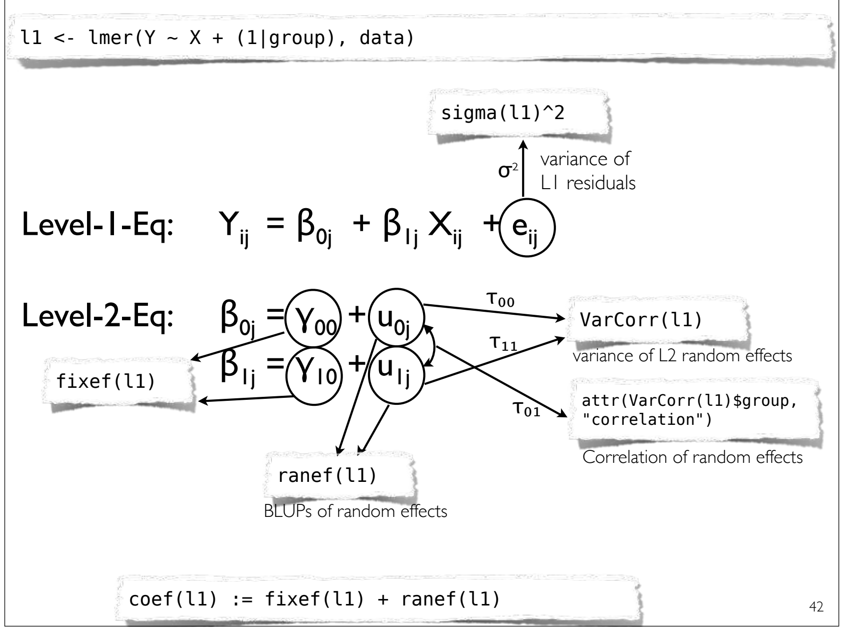

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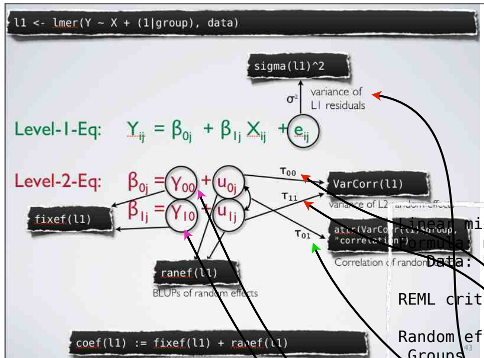

```
Linear mixed model fit by REML ['lmerMod']
Formula: mathach ~ ses + (ses | id)
   Data: HSB

REML criterion at convergence: 46640.4

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept)  4.8285  2.1974
          ses          0.4129  0.6426   0.109
 Residual             36.8302  6.0688
Number of obs: 7185, groups: id, 160

Fixed effects:
            Estimate Std. Error t value
(Intercept)  12.6650     0.1898   66.71
ses           2.3938     0.1181   20.27

Correlation of Fixed Effects:
    (Intr)
ses -0.045
```

Random effects?

Uncertainty vs. variability

- Uncertainty = lack of complete knowledge about a parameter
  ➞ will reduce with increasing *n*
- Variability = underlying differences between groups
  ➞ will not change systematically with changing *n*

```
mathach ~ ses + (ses | id)

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept)  4.8285  2.1974
          ses          0.4129  0.6426   -0.109
 Residual             36.8302  6.0688
Number of obs: 7185, groups: id, 160

Fixed effects:
            Estimate Std. Error t value
(Intercept)  12.6650     0.1898   66.71
ses           2.3938     0.1181   20.27
```

VARIABILITY

UNCERTAINTY

Gelman & Hill (2007), p. 457 45

Uncertainty in inference: How **confident** can we be about an estimate?

```
# Random intercept + random slope model
l2 <- lmer(mathach ~ ses + (ses|id), HSB)

## compute confidence intervals for the fixed (and random) effects
## Three methods:
# "Wald" (only fixed effects)
# deviance "profile" (can fail on some random effects)
# "boot" (takes longest, but is the safest)

# < 1 sec. on a 2012 Macbook Pro, only fixed effects
c1 <- confint(l2, method="Wald")

# ~ 11 sec. on a 2012 Macbook Pro
c2 <- confint(l2, method="profile")

# ~ 94 sec. on a 2012 Macbook Pro
c3 <- confint(l2, method="boot")
```

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Uncertainty in inference: How **confident** can we be about an estimate?

```
mathach ~ ses + (ses | id)

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept)  4.8285  2.1974
          ses          0.4129  0.6426   -0.109
 Residual             36.8302  6.0688
Number of obs: 7185, groups: id, 160

Fixed effects:
            Estimate Std. Error t value
(Intercept)  12.6650     0.1898   66.71
ses           2.3938     0.1181   20.27
```

```
CONFIDENCE INTERVALS FOR:
----------------------------------------

RANDOM STANDARD DEVIATIONS

                 sqrt(tau_00)   tau_01 (as Corr)  sqrt(tau_11)   sigma
Wald             NA             NA                NA             NA
Deviance profile [1.91; 2.51]   NA                [0.08; 0.97]   [5.97; 6.17]
Bootstrap        [1.89; 2.46]   [-0.99; 0.41]     [0.10; 0.98]   [5.97; 6.18]


FIXED EFFECTS

                 Intercept      ses
Wald             [12.29; 13.04] [2.16; 2.63]
Deviance profile [12.29; 13.04] [2.16; 2.63]
Bootstrap        [12.31; 13.06] [2.16; 2.62]
```

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Interpretation of random variance

I FOUND A RANDOM VARIANCE OF 5.56!

OK. GREAT. (?)

- Compute range of plausible values for the fixed parameter
  - fixed effect ± 1.96 * sqrt(random variance)
  - Parameter of 95% of the units is within this predicted range
- Look at *reduction* of random (= unexplained) variance by new L2 predictors.

THIS IS *NOT* A CONFIDENCE INTERVAL!

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1. The Null-Model: Random effects ANOVA
2. L1 predictor with random intercepts
3. L1 predictor with random intercepts and slopes
4. **Adding a L2 predictor: Cross-level interactions**
   a) **L2 moderates intercept**
   b) L2 moderates intercept and slope

L2-predictors: Explaining random variance

- Schools differ in their intercepts and slopes
- Are there characteristics of schools (i.e., L2-predictors), that can explain some of this variance?
  (not in a causal sense, but in a sense of prediction)
- Goal: Reduction of random variance

Terminology

- **Unconditional model:**
  Random coefficients model without L2 predictors
  - I.e., intercepts (and slopes) are allowed to vary randomly, but this variation is not explained.
- **Conditional model:**
  Random coefficients model, where L2 predictors explain (a part) of the random variation
  - I.e., the random effects are conditional upon a L2 variable.


```
lmer(mathach ~ sector + ses + (ses|id), HSB)
```

ANY PREDICTOR, REGARDLESS OF ITS CONCEPTUAL LEVEL, IS SIMPLY ADDED TO THE FORMULA.

„+": CATEGORICAL L2 PREDICTOR MODERATES INTERCEPT, BUT NOT THE SLOPE

Person level (Level 1):

$$mathach_{ij} = \beta_{0j} + \beta_{1j}*ses_{ij} + e_{ij}$$

School level (Level 2):

$$\beta_{0j} = \gamma_{00} + \gamma_{01}*sector_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + u_{1j}$$

PREDICTORS ARE NOT LOCATED ON A CERTAIN „LEVEL" (AS IN THE HLM SOFTWARE) - THEY STAND CREATED EQUAL NEXT TO EACH OTHER

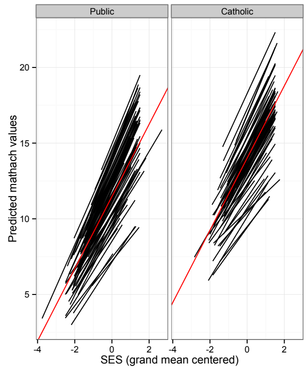
Combined equation:

$$mathach_{ij} = \gamma_{00} + \gamma_{01}*sector_j + u_{0j} + (\gamma_{10} + u_{1j})*ses_{ij} + e_{ij}$$
$$mathach_{ij} = \gamma_{00} + \gamma_{01}*sector_j + \gamma_{10}*ses_{ij} + e_{ij} + u_{0j} + u_{1j}*ses_{ij}$$

```
lmer(mathach ~ sector + ses + (ses|id), HSB)
```



```
b0_Public = 11.47
b0_Catholic = 11.47 + 2.54 = 14.01
b1 =  2.39
```




```
mathach ~ sector + ses + (ses | id)

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept)  3.9644  1.991
          ses          0.4343  0.659    0.550
 Residual             36.8008  6.066
Number of obs: 7185, groups: id, 160

Fixed effects:
               Estimate Std. Error t value
(Intercept)     11.4730     0.2315   49.57
sectorCatholic   2.5407     0.3445    7.37
ses              2.3854     0.1179   20.24
```


Unconditional model

Conditional model:
Random intercept is modeled by „sector“

```
mathach ~ ses + (ses | id)

Random effects:
 Groups   Name        Variance Std.Dev.
 id       (Intercept)  4.8285  2.1974
          ses          0.4129  0.6426
 Residual             36.8302  6.0688
Number of obs: 7185, groups: id, 160

Fixed effects:
            Estimate Std. Error t value
(Intercept)  12.6650     0.1898   66.71
ses           2.3938     0.1181   20.27
```

```
mathach ~ sector + ses + (ses | id)

Random effects:
 Groups   Name        Variance Std.Dev.
 id       (Intercept)  3.9644  1.991
          ses          0.4343  0.659
 Residual             36.8008  6.066
Number of obs: 7185, groups: id, 160

Fixed effects:
               Estimate Std. Error t value
(Intercept)     11.4730     0.2315   49.57
sectorCatholic   2.5407     0.3445    7.37
ses              2.3854     0.1179   20.24
```

```
ICC(id) =
   4.8285 / (4.8285 + 0.4129 + 36.8302) =
   11.5 %
```

```
ICC(id) =
   3.9644 / (3.9644 + 0.4343 + 36.8008) =
   9.6 %
```

THIS IS PSEUDO-R^2 FOR RANDOM INTERCEPTS

```
Proportion variance explained = (4.8285 - 3.9644)/ 4.8285 = 18%
```

18% of previously unexplained random variance in intercepts could be explained by „sector“

1. The Null-Model: Random effects ANOVA
2. L1 predictor with random intercepts
3. L1 predictor with random intercepts and slopes
4. **Adding a L2 predictor: Cross-level interactions**
    a) L2 moderates intercept
    b) **L2 moderates intercept and slope**

```
lmer(mathach ~ sector * ses + (ses|id), HSB)
```

„*": CATEGORICAL L2 PREDICTOR MODERATES INTERCEPT AND SES SLOPE

Person level (Level 1):

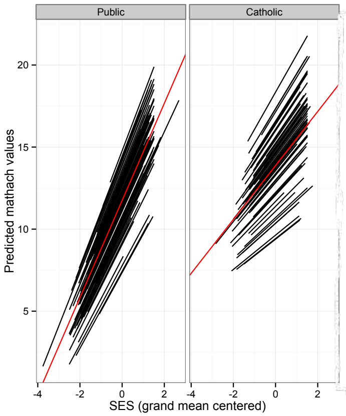
$mathach_{ij} = \beta_{0j} + \beta_{1j}*ses_{ij} + e_{ij}$

School level (Level 2):

$\beta_{0j} = \gamma_{00} + \gamma_{01}*sector_j + u_{0j}$
$\beta_{1j} = \gamma_{10} + \gamma_{11}*sector_j + u_{1j}$

```
lmer(mathach ~ sector * ses + (ses|id), HSB)
```

```
b0_Public = 11.75
b0_Catholic = 11.75 + 2.13 = 13.88
b1_Public =  2.96
b1_Catholic = 2.96 - 1.31 = 1.64
```




```
mathach ~ sector * ses + (ses | id)

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept)  3.82163 1.9549
          ses          0.07585 0.2754   1.000
 Residual             36.78751 6.0653
Number of obs: 7185, groups: id, 160

Fixed effects:
                   Estimate Std. Error t value
(Intercept)         11.7518     0.2318   50.70
sectorCatholic       2.1295     0.3459    6.16
ses                  2.9575     0.1431   20.67
sectorCatholic:ses  -1.3134     0.2156   -6.09
```

Model comparison

- Models *must* be fit to exactly the same data!
- χ^2 test: If the test is significant, the more complex model (with more predictor variables) is significantly better.

```
> anova(l3, l4)

Models:
l3: mathach ~ sector + ses + (ses | id)
l4: mathach ~ sector * ses + (ses | id)

   Df   AIC   BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
l3  7 46611 46660 -23299    46597
l4  8 46579 46634 -23282    46563 34.214      1  4.939e-09 ***
```

MODELING SLOPES BY „SECTOR" SIGNIFICANTLY IMPROVES THE MODEL!

Overview model syntax

```
# random intercept, no predictor (= ANOVA)
lmer(mathach ~ 1 + (1|id), HSB)

# random intercept, L1 predictor
lmer(mathach ~ ses + (1|id), HSB)

# random intercept + random slope, L1 predictor
lmer(mathach ~ ses + (ses|id), HSB)


# only random slope (fixed intercept), L1 predictor
lmer(mathach ~ ses + (0 + ses|id), HSB)

# random intercept + random slope, L1 predictor, L2 predictor,
# cross-level interaction

# sector only affects intercepts
lmer(mathach ~ sector + ses + (ses|id), HSB)

# sector affects intercepts and slopes
lmer(mathach ~ sector * ses + (ses|id), HSB)
```


Terminology: Fixed vs. Random Effects

[...] we briefly review **what is meant by fixed and random effects**. It turns out that different—in fact, incompatible—definitions are used in different contexts. Here we outline **five definitions** that we have seen:

1. Fixed effects are constant across individuals, and random effects vary. For example, in a growth study, a model with random intercepts αi and fixed slope β corresponds to parallel lines for different individuals i, or the model yit = αi + βt. (Kreft and de Leeuw, 1998).

2. Effects are fixed if they are interesting in themselves or random if there is interest in the underlying population. (Searle, Casella and McCulloch, 1992)

3. “When a sample exhausts the population, the corresponding variable is fixed; when the sample is a small (i.e., negligible) part of the population the corresponding variable is random” (Green and Tukey, 1960).

4. “If an effect is assumed to be a realized value of a random variable, it is called a random effect” (LaMotte, 1983).

5. Fixed effects are estimated using least squares (or, more generally, maximum likelihood) and random effects are estimated with shrinkage. This definition is standard in the multilevel modeling literature (e.g., Snijders & Bosker, 1999) and in econometrics.

Gelman (2005, p. 20)

Terminology: Fixed vs. Random Effects

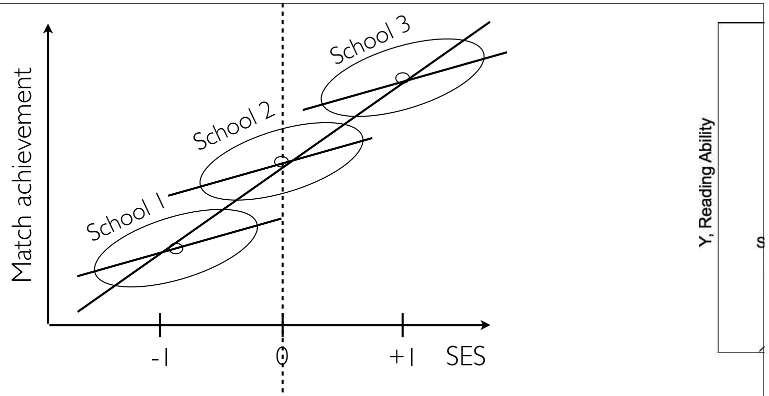
[...] we briefly review **what is meant by fixed and random effects**. It turns out that different—in fact, incompatible—definitions are used in different contexts. Here we outline **five definitions** that we have seen:

1. Fixed effects are constant across individuals, and random effects vary. For example, in a growth study, a model with random intercepts αi and fixed slope β corresponds to parallel lines for different individuals i, or the model yit = αi + βt. (Kreft and de Leeuw, 1998).

We prefer to sidestep the overloaded terms “fixed” and “random” with a cleaner distinction by simply renaming the terms in definition 1 above. We define effects (or coefficients) in a multilevel model as **constant** if they are identical for all groups in a population and **varying** if they are allowed to differ from group to group.

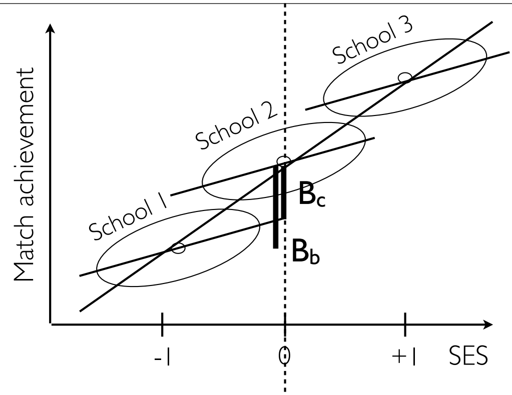
Gelman (2005, p.21)

Centering



Grand-Mean-Centering of SES:
B_c is the difference of pupils from different schools but with equal SES.

Group-Mean-Centering of SES:
B_b is the difference of pupils from different schools with each having an average SES relative to their school.



Grand-Mean-Centering of SES:
B_c is the difference of pupils from different schools but with equal SES.

Group-Mean-Centering of SES:
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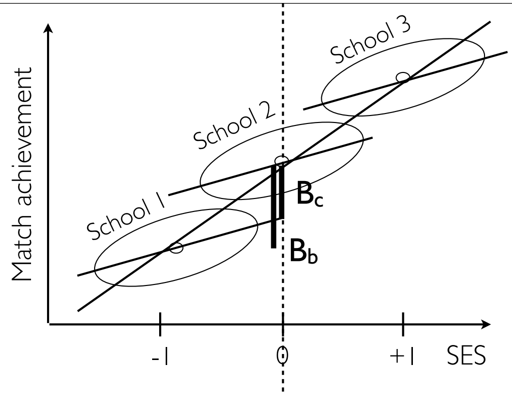

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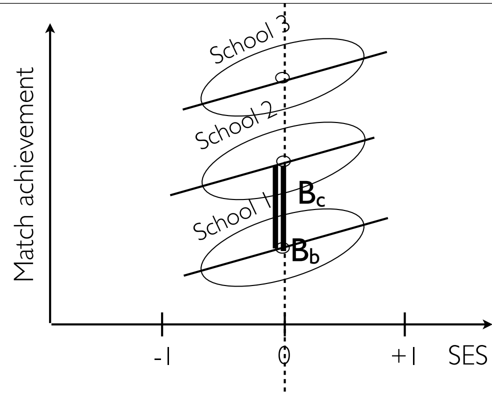
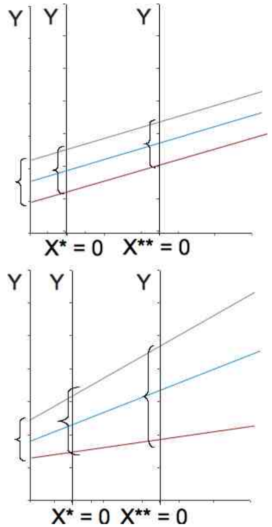


Grand-Mean-Centering of SES:
B_c is the difference of pupils from different schools but with equal SES.

Group-Mean-Centering of SES:
B_b is the difference of pupils from different schools with each having an average SES relative to their school.

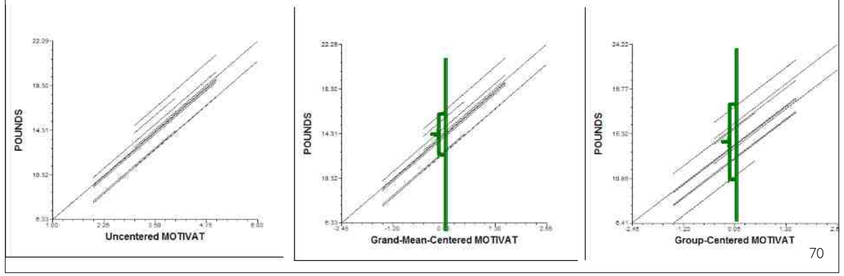
Centering – Consequences on random intercept variance

- Intercept can be interpreted
- For models *without* varying slopes the variance of intercepts for uncentered and grand mean centered predictors are equal
- The variance of intercepts changes when predictors are group-mean centered.
- For models with varying slopes the variance of intercepts changes after centering



Centering – Conclusion

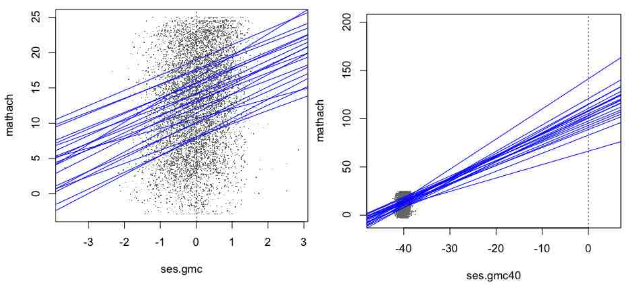
- The centering of level 1 predictors can affect the variance of intercepts τ_{00}.
- This variance is often to be explained by level 2 predictors.
- Therefore, it depends on the centering whether a level 2 predictor has a significant effect or not.



τ_{01}: Correlation between random intercepts and random slopes

- Correlation of 1 is OK (no indicator of a fitting problem)
- Correlation of 1 can be an indicator of *inappropriate centering*:

Shift of ses	τ
0	.02
1	.29
2	.51
3	.66
40	.996



The Disaggregation of Within-Person and Between-Person Effects:

„Persons as Contexts"
aka.
„Centered within context with reintroduction of the mean at level 2: CWC(M)" approach
aka.
„frog-pond model"

Curran, P. J., & Bauer, D. J. (2011). The Disaggregation of Within-Person and Between-Person Effects in Longitudinal Models of Change. Annual Review of Psychology, 62(1), 583–619.

Hoffman, L., & Stawski, R. S. (2009). Persons as contexts: Evaluating between-person and within-person effects in longitudinal analysis. Research in Human Development, 6(2-3), 97–120.

West, S. G., Ryu, E., Kwok, O.-M., & Cham, H. (2011). Multilevel Modeling: Current and Future Applications in Personality Research. Journal of Personality, 79(1), 2–50.

Zhang, Z., Zyphur, M. J., & Preacher, K. J. (2009). Testing Multilevel Mediation Using Hierarchical Linear Models: Problems and Solutions. Organizational Research Methods, 12(4), 695–719.

CWC(M) (aka. CWC2)

Centering within context, with the mean reintroduced at Level 2

- Person-centered predictor now means: „*deviations from the person's usual level*"
- But: removing the persons mean removes all the between-persons variation from the model ➙ loss of information!
- Therefore: Reintroduce the person mean as a Level 2 predictor.
- For better interpretability of the person means, do a grand mean centering

Three ways of centering

A) Grand-mean centering of L1
➙ OK for covariate-model, but confound of within- and between-variation!

B) Grand-mean centering of L1+ person-mean on L2
➙ „contextual effect"
- mean effect, controlling for daily variation
- daily effect, controlling for mean level effect

C) Person-mean centering of L1 + person mean on L2 (CWC2)
➙ „pure between persons effect": **recommended for disaggregation of between- and within effects!**
- The CWC(M) approach is currently regarded as **the best approach** for the disaggregation of within- and between-person effects (Curran & Bauer, 2011). It is valid, as long as the predictor variable is not depended on time (i.e., no systematic growth or decline).

General approach:

1. Compute mean for L1 predictor within L2 unit
2. Within persons: Substract mean from L1 predictor
➙ this is your L1 `.pc` variable („person-centered")
3. Merge computed means with long data frame
➙ this is your L2 `.pm` variable („person-mean")
4. Include `.pc` and `.pm` variable to model

Final model for HSB

- ses.gc = ses group-centered (Level 1)
- ses.gm = ses group mean (Level 2), uncentered
- ses.gmc = ses group mean (Level 2), centered

```
## group-centering of ses:
-----------------------------------------
# compute group means, add to L1 data frame (group mean = .gm)
HSB$ses.gm <- ave(HSB$ses, HSB$id)

# center ses within group (group-centered = .gc)
HSB$ses.gc <- HSB$ses - HSB$ses.gm

# center the L2 predictor for better interpretability
# Be careful: the mean of ses.gm in the long format data frame is *not*
the mean of the group means (because the ses.gm is repeated for each
person within unit, and unequal sample sizes distort the mean)

meanses <- aggregate(ses ~ id, HSB, mean)
# ses.gm.mean stores the mean of group means
ses.gm.mean <- mean(meanses$ses)
HSB$ses.gmc <- HSB$ses.gm - ses.gm.mean
```

Final model for HSB: Unconditional

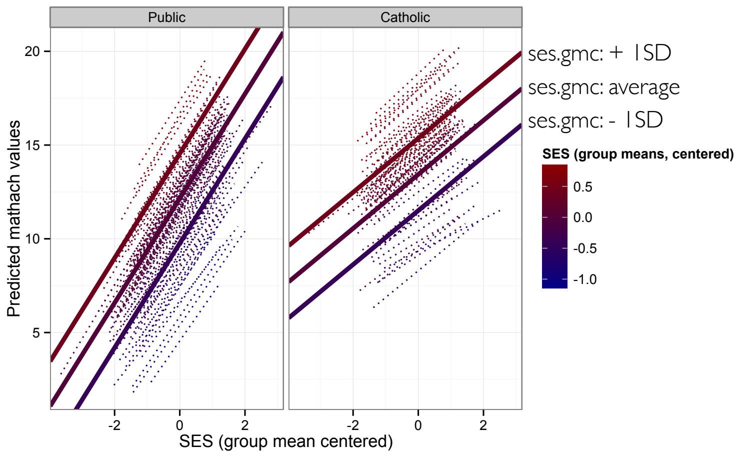
- ses.gc = ses group-centered (Level 1)
- ses.gmc = ses group mean (Level 2), centered

```
mathach ~ ses.gc + ses.gmc + (ses.gc | id)

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept)  2.68004 1.6371
          ses.gc       0.03924 0.1981   -1.00
 Residual             37.00631 6.0833
Number of obs: 7185, groups: id, 160

Fixed effects:
            Estimate Std. Error t value
(Intercept)  12.6805     0.1491   85.04
ses.gc        2.1883     0.1098   19.93
ses.gmc       5.9211     0.3584   16.52
```


Final model for HSB: Conditional

```
mathach ~ sector * ses.gc + sector * ses.gmc + (ses.gc | id)

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept)  2.3455  1.531
          ses.gc       0.2704  0.520    0.25
 Residual             36.7102  6.059
Number of obs: 7185, groups: id, 160

Fixed effects:
                       Estimate Std. Error t value
(Intercept)             12.1904     0.2040   59.76
sectorCatholic           1.2717     0.3048    4.17
ses.gc                   2.7873     0.1558   17.89
ses.gmc                  5.7770     0.5052   11.44
sectorCatholic:ses.gc   -1.3463     0.2347   -5.74
sectorCatholic:ses.gmc  -1.1225     0.7334   -1.53
```


Final model for HSB: Conditional



DEMO!

The BEER data set

This is a simulated data set. For simulation code, see material.

The BEER project

- Alcohol use is positively correlated with IQ (Batty et al., 2008; Wilmoth, 2012)
- Alcohol use leads to impaired cognitive abilities (Tzambazis & Stough, 2000)

BETWEEN PERSONS!

WITHIN PERSONS!

- Design:
  - 200 participants, each measured at 6 to 12 occasions, several days apart
  - IV: Beer intake in the last 12 hours
  - DV: Score in an IQ test

The idea for this study came from Kievit & Epskamp (2013)

The BEER project

Hypotheses

- On a *within-person* level, more beer intake in the last 12 hours leads to less cognitive ability in an IQ test
  - This effect is moderated by sex: the adverse impact of beer is more pronounced for females
- On a *between-person* level, more beer intake is associated with higher IQs
- → Search for a ecological fallacy!

PLEASE WRITE THE MLM MODEL FORMULA AS A CWC(M) MODEL!

The BEER project

Model formula

- On a *within-person* level, more beer intake in the last 24 hours leads to less cognitive ability in an IQ test
- This effect is moderated by sex: the adverse impact of beer is more pronounced for females
- On a *between-person* level, more beer intake is associated with higher IQs

Level 1: Repeated measurement (within person model):

$IQ_{ti} = \beta_{0i} + \beta_{1i} * BEER.pc_{ti} + e_{ti}$

Level 2: Between-person model

$\beta_{0i} = \gamma_{00} + \gamma_{01} * BEER.pmc_i + \gamma_{02} * SEX_i + u_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11} * SEX_i + u_{1i}$

Combined equation:

$IQ_{ti} = [\gamma_{00} + \gamma_{01} * BEER.pmc_i + \gamma_{02} * SEX_i + u_{0i}] + [(\gamma_{10} + \gamma_{11} * SEX_i + u_{1i}) * BEER.pc_{ti}] + e_{ti}$

i = index of person;
t = index of measurement occasion;
.pc = person-centered;
.pmc = person mean, centered

Compute new variables

```
# .pm = person mean
df$beer.pm <- ave(df$beer, df$pid)

# .pc = person-centered
df$beer.pc <- df$beer - df$beer.pm

# .pmc = person-mean, centered (on L1)
MEANBEER <- mean(aggregate(beer~pid, df, mean)[, 2])
df$beer.pmc <- df$beer.pm - MEANBEER
```

NOW: WRITE THE LME4 MODEL SYNTAX FOR INCREASINGLY COMPLEX MODELS!

NULL MODEL → L1 PREDICTOR → L2 PREDICTOR(S)

The BEER project

lme4 syntax

Level 1: Repeated measurement (within person model):

$IQ_{ti} = \beta_{0i} + \beta_{1i} * BEER.pc_{ti} + e_{ti}$

Level 2: Between-person model

$\beta_{0i} = \gamma_{00} + \gamma_{01}*BEER.pmc_i + \gamma_{02}*SEX_i + u_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}*SEX_i + u_{1i}$

i = index of person; t = index of measurement occasion; .pc = person-centered; .pm = person mean

```
lmer(IQ ~ 1 + (1|pid), df)                  # null model
lmer(IQ ~ beer.pc + (1|pid), df)            # L1 predictor, random I
lmer(IQ ~ beer.pc + (beer.pc|pid), df)      # L1 pred., random I + S

# L1 pred. + L2 pred., random I + S
lmer(IQ ~ beer.pmc + beer.pc + (beer.pc|pid), df)

# sex should moderate the slope of beer.pc
lmer(IQ ~ beer.pmc + sex*beer.pc + (beer.pc|pid), df)
```


Let's look a the data ...

Results

```
Formula: IQ ~ sex * beer.pc + beer.pmc + (beer.pc | pid)

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 pid      (Intercept) 69.881   8.359
          beer.pc      1.892   1.376    0.040
 Residual             69.213   8.319
Number of obs: 1782, groups: pid, 200

Fixed effects:
                  Estimate Std. Error t value
(Intercept)       100.3903     0.8756  114.65
sexfemale           1.5717     1.2506    1.26
beer.pc            -3.0929     0.3440   -8.99
beer.pmc            3.8863     0.3363   11.56
sexfemale:beer.pc  -5.6655     0.4783  -11.85
```

Combined equation:

$IQ_{ti} =$
$[\gamma_{00} + \gamma_{01}*BEER.pmc_i + \gamma_{02}*SEX_i] +$
$[(\gamma_{10} + \gamma_{11}*SEX_i)* BEER.pc_{ti}]$

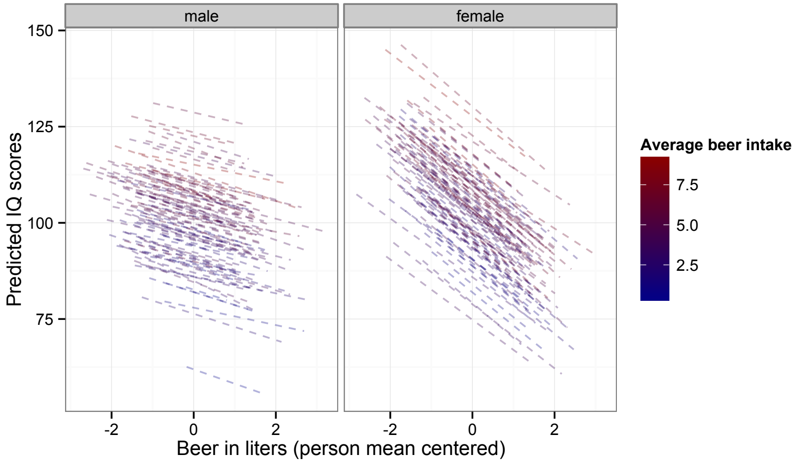
Bootstrapped confidence intervals

	2.5 %	97.5 %
sd_(Intercept)|pid	7.38	9.23
cor_beer.pc.(Intercept)|pid	-.35	1.00
sd_beer.pc|pid	.28	2.07
sigma	8.03	8.64
(Intercept)	98.68	102.06
sexfemale	-.80	3.83
beer.pc	-3.83	-2.40
beer.pmc	3.21	4.53
sexfemale:beer.pc	-6.61	-4.75

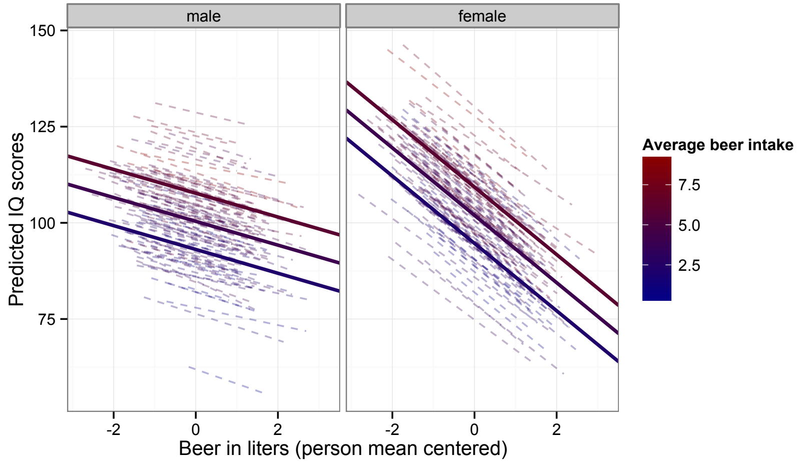
RANDOM EFFECTS

FIXED EFFECTS

Plot: **Person-centered** on X



Plot: **Person-centered** on X



Deconstruct the equation: Find person 101!

```
> df[df$pid==101, ]

pid  sex  beer IQ beer.pm  beer.pmc
101 male  0.0 57   0.17     -3.88
101 male  0.0 59   0.17     -3.88
101 male  0.0 71   0.17     -3.88
101 male  0.0 46   0.17     -3.88
   ...
```

```
> fixef(l2)

(Intercept)         100.390332
sexfemale             1.571684
beer.pc              -3.092936
beer.pmc              3.886274
sexfemale:beer.pc    -5.665495
```

(γ_{00}, γ_{02}, γ_{10}, γ_{01}, γ_{11})

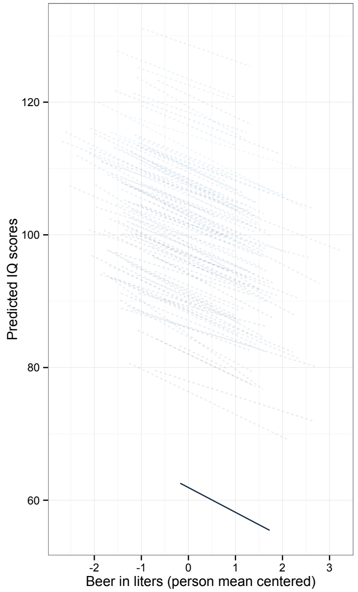
```
> ranef(l2)$pid["101", ]

    (Intercept)    beer.pc
101   -23.38414 -0.6407813
```

Combined equation:

$IQ_{ti} = [\gamma_{00} + u_{0i} + \gamma_{01}*BEER.pmc_i + \gamma_{02}*SEX_i] +$ ← INTERCEPT

$[(\gamma_{10} + u_{1i} + \gamma_{11}*SEX_i)* BEER.pc_{ti}]$ ← SLOPE

$IQ_{101} = [100.39 - 23.38 + 3.89*(-3.88) + 1.57*0] +$
$[(-3.09 - 0.64 - 5.67*0)*BEER$
$= 61.91$
$- 3.73 * beer$



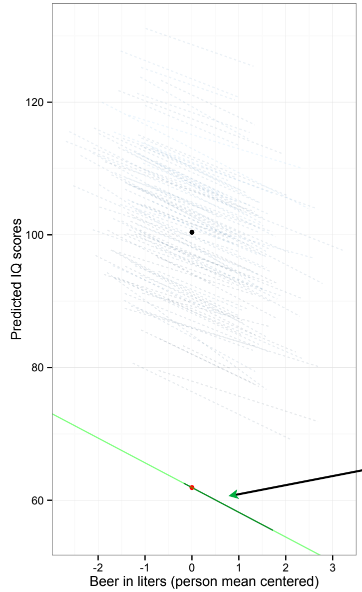

```
> fixef(l2)

(Intercept)        100.390332
sexfemale            1.571684
beer.pc             -3.092936
beer.pmc             3.886274
sexfemale:beer.pc   -5.665495
```

γ_{00}, γ_{02}, γ_{10}, γ_{01}, γ_{11}

```
> ranef(l2)$pid["101", ]

    (Intercept)    beer.pc
101   -23.38414 -0.6407813
```

IQ_{101} = 61.91 - 3.73 * beer

93



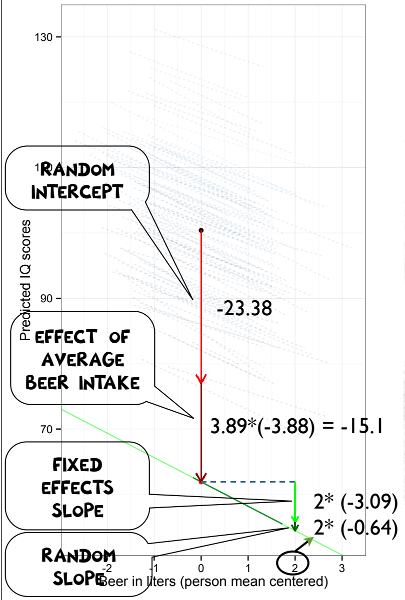

```
> fixef(l2)

(Intercept)        100.390332
sexfemale            1.571684
beer.pc             -3.092936
beer.pmc             3.886274
sexfemale:beer.pc   -5.665495
```

γ_{00}, γ_{02}, γ_{10}, γ_{01}, γ_{11}

```
> ranef(l2)$pid["101", ]

    (Intercept)    beer.pc
101   -23.38414 -0.6407813
```

IQ_{101} = 61.91 - 3.73 * beer

94




```
> fixef(l2)

(Intercept)        100.390332
sexfemale            1.571684
beer.pc             -3.092936
beer.pmc             3.886274
sexfemale:beer.pc   -5.665495
```

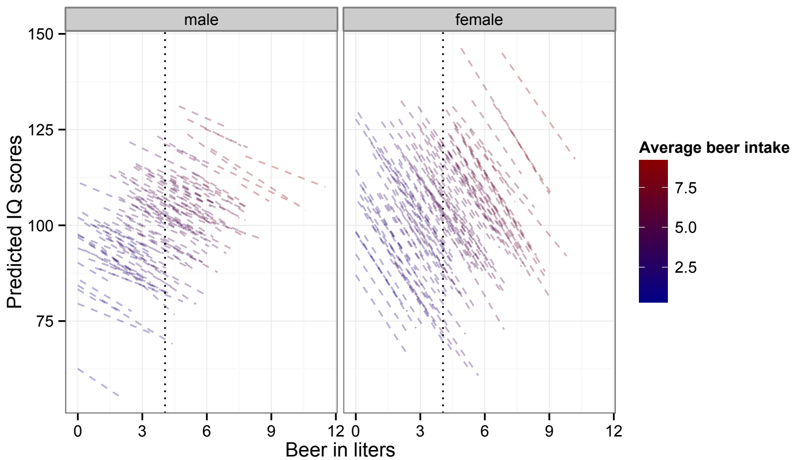
γ_{00}, γ_{02}, γ_{10}, γ_{01}, γ_{11}

```
> ranef(l2)$pid["101", ]

    (Intercept)    beer.pc
101   -23.38414 -0.6407813
```

IQ_{101} = 61.91 - 3.73 * beer

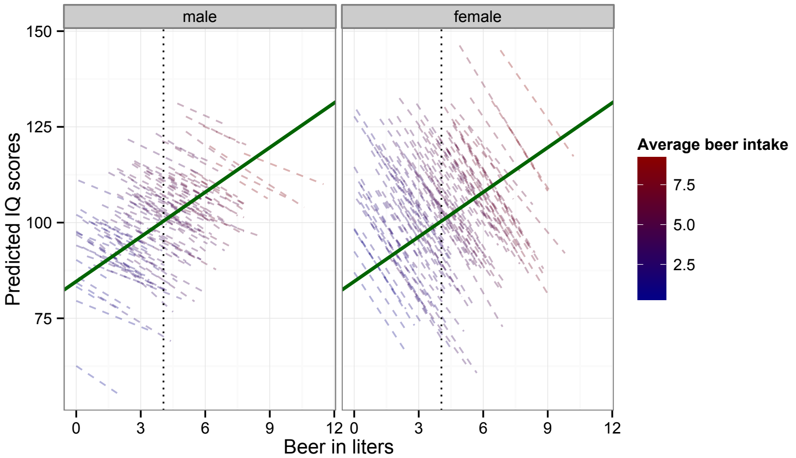
95

Plot: **Raw** on X

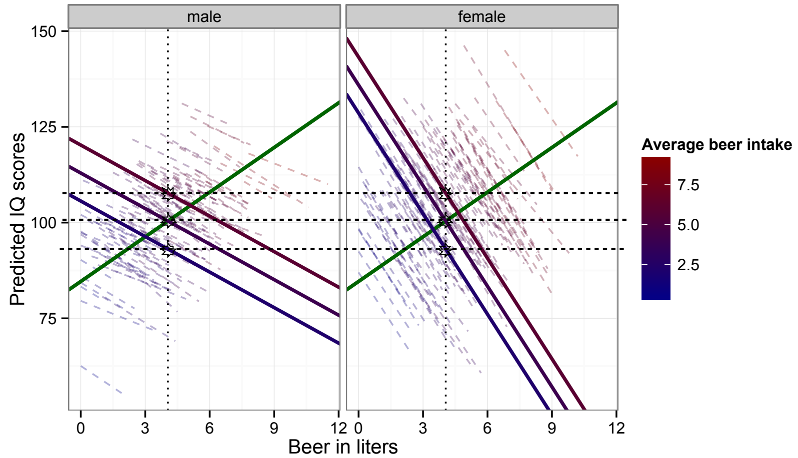


96

Plot: **Raw** on X, now including between-persons regression line



Plot: **Raw** on X, now including between-persons and within-persons regression lines



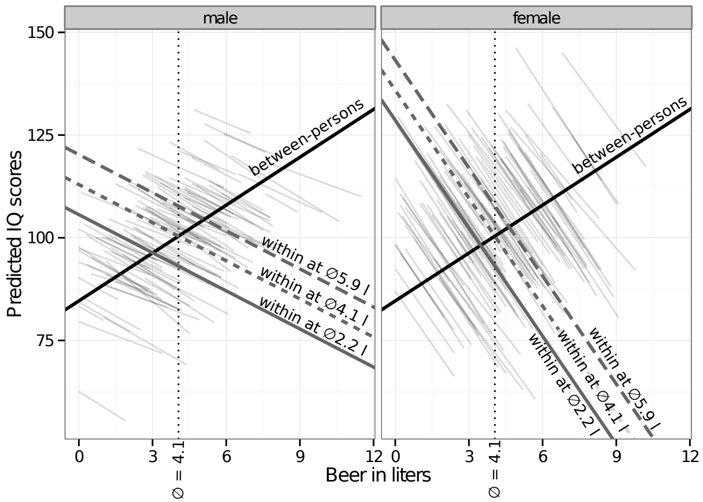


Figure 1. The vertical dotted line shows the mean person-average beer intake (4.1 liter), thin grey lines are individual regression lines. The solid black line shows the between-persons regression slope, the three dark-gray regression lines show within-persons regressions at -1, 0, and +1 SD of average beer intake.

WE ALREADY KNOW: WOMEN ARE MORE IMPAIRED BY BEER INTAKE THAN MEN (ON THE WITHIN-PERSON LEVEL).

WE ALREADY KNOW: PERSONS WITH HIGHER BEER INTAKE (ON AVERAGE, BETWEEN-PERSON LEVEL) HAVE A HIGHER IQ.

THAT MEANS, THE WITHIN-PERSON BEER EFFECT IS MORE PRONOUNCED FOR WOMEN.

IS THE BETWEEN-PERSON BEER EFFECT ALSO MORE PRONOUNCED FOR WOMEN?

TEST FOLLOWING HYPOTHESIS WITHIN THE FULL MODEL: „THE POSITIVE EFFECT OF AVERAGE BEER INTAKE BEER ON IQ IS MORE PRONOUNCED FOR WOMEN"

```
lmer(IQ ~ sex*beer.pc + sex*beer.pm + (1|pid), df)

# RESULTS

Random effects:
 Groups   Name        Variance Std.Dev.
 pid      (Intercept) 69.20    8.318
 Residual             71.03    8.428
Number of obs: 1782, groups: pid, 200

Fixed effects:
                  Estimate Std. Error t value
(Intercept)        82.6460     2.0563   40.19
sexfemale           5.7506     3.0147    1.91
beer.pc            -3.0709     0.3112   -9.87
beer.pm             4.3822     0.4665    9.39
sexfemale:beer.pc  -5.6354     0.4315  -13.06
sexfemale:beer.pm  -1.0215     0.6710   -1.52
```

SEX DOES NOT SIGNIFICANTLY MODERATE THE EFFECT OF AVERAGE BEER INTAKE.

Advanced modeling issues

Where are my *p* values for fixed effects?

I would be happy to re-institute p-values for fixed effects in the summary and anova methods for lmer objects using a denominator degrees of freedom based on the trace of the hat matrix or the rank of Z:X if others will volunteer to respond to the “these answers are obviously wrong because they don’t agree with <whatever> and the idiot who wrote this software should be thrashed to within an inch of his life” messages. I don’t have the patience.

Douglas Bates on http://rwiki.sciviews.org/doku.php?id=guides:lmer-tests

Where are my *p* values for fixed effects?

- Current advice for Gaussian LMMs (might change!): Use KRmodcomp() form the pbkrtest package
  ➙ Nice interface: mixed() function from the afex package

lme4 +
mixed() function (afex package):

```
> mixed(pounds ~ treat*motivatuc + (motivatuc|group), d0)
Fitting 5 lmer() models:
[.....]
Obtaining 4 p-values:
[....]
           Effect     stat ndf     ddf F.scaling p.value
1     (Intercept) 492.9404   1 37.8498         1  0.0000
2           treat   1.2818   1 37.9209         1  0.2647
3       motivatuc  62.8472   1 33.5328         1  0.0000
4 treat:motivatuc  10.9496   1 32.4762         1  0.0023
```

HLM software:

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	14.566701	0.656051	22.204	38	<0.001
TREAT, γ_{01}	0.959466	0.847395	1.132	38	0.265
For MOTIVAT slope, β_1					
INTRCPT2, γ_{10}	2.296567	0.287367	7.992	38	<0.001
TREAT, γ_{11}	1.244499	0.372442	3.341	38	0.002

SE for random variances?

<rant>
Some software, notably SAS PROC MIXED, does produce standard errors for the estimates of variances and covariances of random effects. In my opinion this is more harmful than helpful. The only use I can imagine for such standard errors is to form confidence intervals or to evaluate a z-statistic or something like that to be used in a hypothesis test. However, those uses require that the distribution of the parameter estimate be symmetric, or at least approximately symmetric, and we know that the distribution of the estimate of a variance component is more like a scaled chi-squared distribution which is anything but symmetric. It is misleading to attempt to summarize our information about a variance component by giving only the estimate and a standard error of the estimate.
</rant>

Douglas Bates (2006), https://stat.ethz.ch/pipermail/r-help/2006-July/109308.html

Testing random effects for significance

- Do I need random coefficients (e.g. random slopes), or can I leave them fixed?
  - No easy answer, no accepted conclusion yet ... see: http://glmm.wikidot.com/faq
  - Does it make sense to fix it?
- Problem: Testing a variance for value zero lies at the boundary of the parameter space (0 is the minimum for variances). Therefore, the assumptions for LR-test do not hold.
- Bottom line:
  - Use anova(), and be aware that the *p* value is a conservative estimate (up to 2x of the actual *p* value)
  - An exact test for a single random effect provides the package RLRsim

Testing random effects for significance: ML vs. REML

- „Note that deviances obtained under REML can be used only if the two models compared have the same fixed parts and differ only in their random parts. If this is not the case, **the deviance obtained using Full ML should be used instead**.“ (Bryk & Raudenbush, http://www.ssicentral.com/hlm/help6/faq/Full_ML_vs_Restricted_ML.pdf)
- „The profiled REML criterion has a term that depends on the model matrix X for the fixed-effects parameters. If you change the definition of the fixed-effects you will change the value of that criterion in a systematic way that does not depend on how well the respective models fit the observed data. **Thus, differences in the REML criterion are not a meaningful way to compare two models that differ in the definition of the fixed-effects**“ (Douglas Bates, https://stat.ethz.ch/pipermail/r-sig-mixed-models/2008q3/001328.html)
- → anova() function of lme4 automatically employs ML, even if the model has been fit in REML. That means **anova() can be safely used**, both for changes in fixed and random parts of the model.

Correlated vs. uncorrelated intercept and slopes

```
lmer(mathach ~ ses + (1 + ses|id), HSB)      # Correlated I and S

Random effects:
 Groups   Name        Std.Dev. Corr
 id       (Intercept) 2.1974
          ses         0.6426   -0.11
 Residual             6.0688

Fixed Effects:
(Intercept)          ses
     12.665        2.394
```

```
lmer(mathach ~ ses + (1|id) + (0 + ses|id), HSB)  # Uncorrelated I and S

Random effects:
 Groups   Name        Std.Dev.
 id       (Intercept) 2.2029
 id.1     ses         0.6512
 Residual             6.0681

Fixed Effects:
(Intercept)          ses
     12.653        2.395
```

WHY SHOULD THEY BE UNRELATED?

Where is my R^2?

- „no recipe will have all of the properties of R^2 in the simple linear model case"
  ➞ different MLM-flavors of R^2 highlight different aspects
  (B. Bolker, http://glmm.wikidot.com/faq)
- e.g., pseudo-R^2 = cor(observed, fitted)
  - Ignores multilevel structure
  - Can lead to negative pseudo-R^2
  - Should not be used
- Probably best approach, currently: „A general and simple method for obtaining R2 from generalized linear mixed-effects models" (implemented in MuMIn package („Multi-Model Inference"), function: `r.squaredGLMM`)
  - R2m = Marginal R^2 represents the variance explained by fixed factors
  - R2c = Conditional R^2 is interpreted as variance explained by both fixed and random factors (i.e . the entire model)

https://jslefche.wordpress.com/2013/03/13/r2-for-linear-mixed-effects-models/
Nakagawa, S., & Schielzeth, H. (2013). A general and simple method for obtaining R2 from generalized linear mixed-effects models. Methods in Ecology and Evolution, 4, 133–142. doi:10.1111/j.2041-210x.2012.00261.x

Statistical Power

- What is the effect of interest?
  - Fixed effects? On L1 or on L2?
  - Random variances?
  - Cross-level interactions?
  - ➞ each one calls for a different optimal allocation of samples to units
- Typically, increasing higher level units gives more power than adding more persons within each group, but ...
- Do not trust rules of thumb (like 30/30 rule, 50/20 rule, etc.). Better use Monte-Carlo-simulations with tailored parameters
  - PinT is a program for Power analysis IN Two-level designs (Snijders, 2005, http://www.stats.ox.ac.uk/~snijders/multilevel.htm)
  - Power calculator to detect cross-level interactions (Mathieu et al., 2012, http://mypage.iu.edu/~haguinis/crosslevel.html)

Snijders, Tom A.B. (2005). *Power and Sample Size in Multilevel Linear Models*. In: B.S. Everitt and D.C. Howell (eds.), Encyclopedia of Statistics in Behavioral Science. Volume 3, 1570–1573. Chicester: Wiley.
Scherbaum, C. A., & Ferreter, J. M. (2008). Estimating Statistical Power and Required Sample Sizes for Organizational Research Using Multilevel Modeling. *Organizational Research Methods, 12(2)*, 347–367. doi:10.1177/1094428107308906

Slopes: Random or not random?

- Set **all** slopes to random that potentially can vary between units
- "In the typical case in which fixed-effect slopes are of interest, models without random intercepts will have reduced power, while models without random slopes will exhibit an increased Type I error rate. This suggests that LMEMs with *maximal random effects* structure have the best potential to produce *generalizable results*." (Barr et al., 2013, p. 262)"
- Convergence problems? Think about fixed intercepts + random slopes!

Barr, D. J. (2013). Random effects structure for testing interactions in linear mixed-effects models, 1–2. doi:10.3389/fpsyg.2013.00328/full
Barr, D. J., Levy, R., Scheepers, C., & Tily, H. J. (2013). Random effects structure for confirmatory hypothesis testing: Keep it maximal. *Journal of Memory and Language, 68(3)*, 255–278. doi:10.1016/j.jml.2012.11.001

Three-level longitudinal models

NOW „TIME" IS A PREDICTOR!

Three-level longitudinal models

Level-1: Repeated measurement within person

$MATH_{tij} = \pi_{0ij} + \pi_{1ij}*\mathbf{YEAR} + e_{tij}$

YEAR = 0, 1, 2, 3
I.E., INTERCEPT = BASELINE

Level 2: Person characteristics

$\pi_{0ij} = \beta_{00j} + \beta_{01j}*\mathbf{GENDER}_{ij} + r_{0ij}$

$\pi_{1ij} = \beta_{10j} + \beta_{11j}*\mathbf{GENDER}_{ij} + r_{1ij}$

GENDER:
0 = FEMALE
1 = MALE

Level-3: School characteristics

$\beta_{00j} = \gamma_{000} + \gamma_{001}*\mathbf{HISP.SCH}_{1j} + u_{00j}$

$\beta_{10j} = \gamma_{100} + \gamma_{101}*\mathbf{HISP.SCH}_{1j} + u_{10j}$

HISP.SCH =
PROPORTION OF HISPANICS IN SCHOOL
(FROM 0 TO 1)

Data set from http://pages.uoregon.edu/stevensj/HLM/data/

Three-level models

Level-1: Repeated measurement within person

$MATH_{tij} = \pi_{0ij} + \pi_{1ij}*YEAR + e_{tij}$

Level 2: Person characteristics

$\pi_{0ij} = \beta_{00j} + \beta_{01j}*GENDER_{ij} + \mathbf{r_{0ij}}$

$\pi_{1ij} = \beta_{10j} + \beta_{11j}*GENDER_{ij} + \mathbf{r_{1ij}}$

INTERCEPT IS ALLOWED TO VARY BETWEEN PERSONS

SLOPE IS ALLOWED TO VARY BETWEEN PERSONS

Level-3: School characteristics

$\beta_{00j} = \gamma_{000} + \gamma_{001}*HISP.SCH_{1j} + \mathbf{u_{00j}}$

$\beta_{10j} = \gamma_{100} + \gamma_{101}*HISP.SCH_{1j} + \mathbf{u_{10j}}$

SCHOOL-AVERAGE INTERCEPTS ARE ALLOWED TO VARY AROUND THE GRAND MEAN

SCHOOL-AVERAGE SLOPES ARE ALLOWED TO VARY AROUND THE GRAND MEAN

Three-level models

LEARNING RATE:
MATH ~ YEAR

Level-1: Repeated measurement within person

$MATH_{tij} = \pi_{0ij} + \pi_{1ij}*\mathbf{YEAR} + e_{tij}$

PERSON-INTERCEPT IS PREDICTED BY GENDER

Level 2: Person characteristics

$\pi_{0ij} = \beta_{00j} + \beta_{01j}*\mathbf{GENDER}_{ij} + r_{0ij}$

$\pi_{1ij} = \beta_{10j} + \beta_{11j}*\mathbf{GENDER}_{ij} + r_{1ij}$

PERSON-SLOPE (LEARNING RATE) IS PREDICTED BY GENDER

Level-3: School characteristics

$\beta_{00j} = \gamma_{000} + \gamma_{001}*\mathbf{HISP.SCH}_{1j} + u_{00j}$

$\beta_{10j} = \gamma_{100} + \gamma_{101}*\mathbf{HISP.SCH}_{1j} + u_{10j}$

SCHOOL-INTERCEPTS ARE PREDICTED BY PROP. OF HISPANICS

SCHOOL-SLOPES ARE PREDICTED BY PROP. OF HISPANICS

Three-level models

DO PUPILS LEARN SOMETHING OVER THE YEARS?

LEARNING RATE:
MATH ~ YEAR

Level-1: Repeated measurement within person

MATH

ARE MALES SMARTER AT THE BEGINNING?

PERSON-INTERCEPT IS PREDICTED BY GENDER

Level 2: Person characteristics

π_{0ij}

π_{1ij}

DO MALES LEARN FASTER?

PERSON-SLOPE (LEARNING RATE) IS PREDICTED BY GENDER

Level-3: School characteristics

β_{00j}

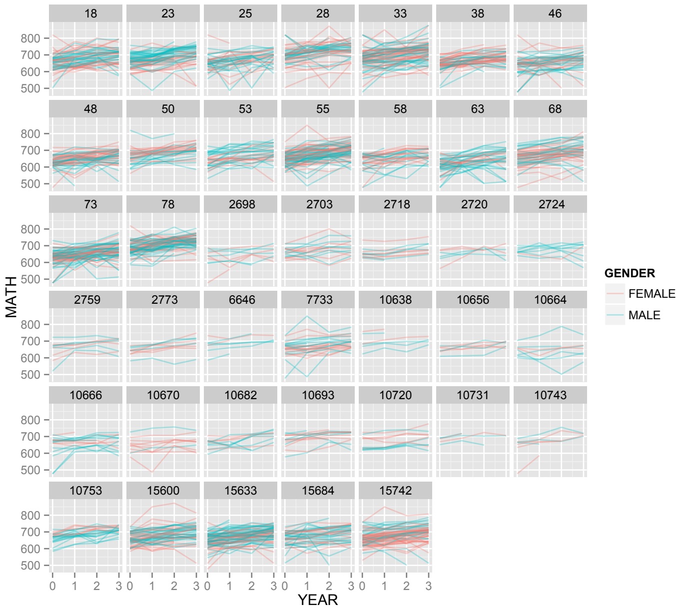
β

DO SCHOOLS WITH MANY HISPANICS START WITH LOWER SCORES?

SCHOOL-INTERCEPTS ARE PREDICTED BY PROP. OF HISPANICS

HOW IS LEARNING RATE INFLUENCED BY THE PROPORTION OF HISPANICS?

SCHOOL-SLOPES ARE PREDICTED BY PROP. OF HISPANICS



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Unique id?? IMPORTANT!

- Check if person IDs are unique!
- „implicit nesting“: If person IDs repeat between schools, lme4 assumes that the same pupil is in different schools! (➙ a cross-classified structure)

```
isNested(d2$SCHID, d2$CASEID)
```

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Results from the three-level model

```
MATH ~ HISP.SCH * YEAR + GENDER * YEAR +
(YEAR | CASEID) + (YEAR | SCHID)

Random effects:
 Groups   Name        Variance  Std.Dev.
 CASEID   (Intercept) 1435.5826 37.8891
          YEAR          34.5548  5.8783
 SCHID    (Intercept)   98.6313  9.9313
          YEAR           0.8522  0.9231
 Residual              508.0818 22.5407
Number of obs: 5096,
groups: CASEID, 1497; SCHID, 40

Fixed effects:
                Estimate Std. Error t value
(Intercept)     680.2546     4.7329  143.73
HISP.SCH        -52.1449     8.4653   -6.16
YEAR             14.3165     0.9001   15.91
GENDERMALE       -2.1197     2.2259   -0.95
HISP.SCH:YEAR    -4.8049     1.5376   -3.12
YEAR:GENDERMALE   1.6398     0.6701    2.45
```

Level-1: Repeated measurement within person

$MATH_{tij} = \pi_{0ij} + \pi_{1ij} * \mathbf{YEAR} + \mathbf{e_{tij}}$

Level 2: Person characteristics

$\pi_{0ij} = \beta_{00j} + \beta_{01j} * \mathbf{GENDER_{ij}} + \mathbf{r_{0ij}}$

$\pi_{1ij} = \beta_{10j} + \beta_{11j} * \mathbf{GENDER_{ij}} + \mathbf{r_{1ij}}$

Level-3: School characteristics

$\beta_{00j} = \gamma_{000} + \gamma_{001} * \mathbf{HISP.SCH_{1j}} + \mathbf{u_{00j}}$

$\beta_{10j} = \gamma_{100} + \gamma_{101} * \mathbf{HISP.SCH_{1j}} + \mathbf{u_{10j}}$

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```
MATH ~ HISP.SCH * YEAR + GENDER * YEAR +
(YEAR | CASEID) + (YEAR | SCHID)

Random effects:
 Groups   Name        Variance  Std.Dev.
 CASEID   (Intercept) 1435.5826 37.8891
          YEAR          34.5548  5.8783
 SCHID    (Intercept)   98.6313  9.9313
          YEAR           0.8522  0.9231
 Residual              508.0818 22.5407
Number of obs: 5096,
groups: CASEID, 1497; SCHID, 40

Fixed effects:
                Estimate Std. Error t value
(Intercept)     680.2546     4.7329  143.73
HISP.SCH        -52.1449     8.4653   -6.16
YEAR             14.3165     0.9001   15.91
GENDERMALE       -2.1197     2.2259   -0.95
HISP.SCH:YEAR    -4.8049     1.5376   -3.12
YEAR:GENDERMALE   1.6398     0.6701    2.45
```

Level-1: Repeated measurement within person

$MATH_{tij} = \pi_{0ij} + \pi_{1ij}*YEAR + e_{tij}$

Level 2: Person characteristics

$\pi_{0ij} = \beta_{00j} + \beta_{01j}*GENDER_{ij} + r_{0ij}$

$\pi_{1ij} = \beta_{10j} + \beta_{11j}*GENDER_{ij} + r_{1ij}$

Level-3: School characteristics

$\beta_{00j} = \gamma_{000} + \gamma_{001}*HISP.SCH_{1j} + u_{00j}$

$\beta_{10j} = \gamma_{100} + \gamma_{101}*HISP.SCH_{1j} + u_{10j}$

Reordering the fixed effects

```
                    Estimate Std. Error t value

Model for initial status:
     Starting point for female pupils in schools with 0% hispanics:
          (Intercept)     680.2546     4.7329  143.73

     Model for ethno gap in initial status:
          HISP.SCH        -52.1449     8.4653   -6.16

     Model for gender gap in initial status:
          GENDERMALE       -2.1197     2.2259   -0.95

Model for learning rate:
     Learning rate of female pupils in schools with 0% hispanics:
          YEAR             14.3165     0.9001   15.91

     Model for influence of gender on learning rate:
          YEAR:GENDERMALE   1.6398     0.6701    2.45

     Model for influence of school hispanics prop on learning rate:
          HISP.SCH:YEAR    -4.8049     1.5376   -3.12
```


Reordering the fixed effects

```
Model for initial status:
     Starting point for female pupils in schools with 0% hispanics:
          (Intercept)     680.2546

     Model for ethno gap in initial status:
          HISP.SCH        -52.1449

     Model for gender gap in initial status:
          GENDERMALE       -2.1197

Model for learning rate:
     Learning rate of female pupils in schools with 0% hispanics:
          YEAR             14.3165

     Model for influence of gender on learning rate:
          YEAR:GENDERMALE   1.6398

     Model for influence of school hispanics prop on learning rate:
          HISP.SCH:YEAR    -4.8049
```

„Hispanic" schools start lower ...

Males start lower (but not significantly)

Female pupils in non-hispanic schools learn something (14 point gain / year)

Males learn faster (14.3 + 1.6 = 15.9 points / year)

Pupils in 100% „Hispanic" schools learn slower: Females: 14.3 - 4.8 = 9.5 points /year)

GLMMs: Multilevel logistic regression

Generalized linear mixed models (GLMMs)

Once upon a time „linear model“ meant **y = Xβ + e**. In fact, **y = Xβ + e**, where **e** was assumed to have a Gaussian distribution, was routinely referred to as the „general“ linear model.

Once upon a time is no more.

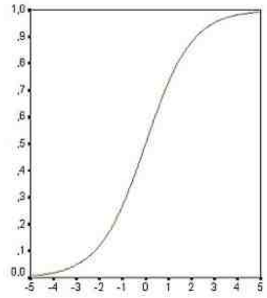
By contemporary standards, **y = Xβ + e** is only a special case. Calling it „general“ seems quaint. It is certainly misleading. [...]

In other words, as of 2012, „linear model“ means „generalized linear mixed model“ (GLMM) and all other linear mixed models are subsumed as special cases of the GLMM.

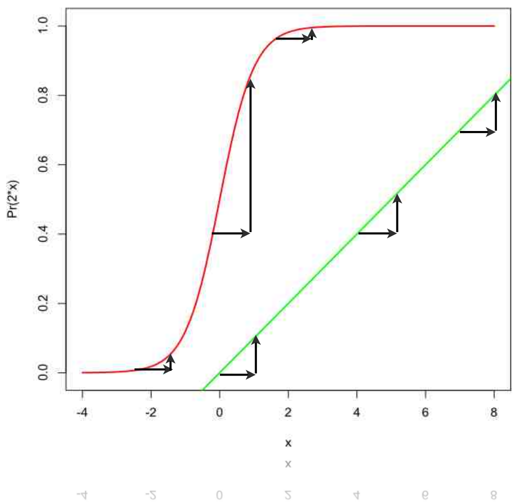
Stroup (2013)

Logistic Regression

- Binary dependent variable (0 and 1)
- $Pr(Y_i = 1) = logit^{-1}\ (\ \beta_1 * X_1 + \beta_2 * X_2 + ...)$
  - predictor = linear combination (as in standard regression)
  - link function: inverse logit log(x/(1-x))
  - transforms predictor values from -∞ bis ∞ to values between 0 in 1



Logistic Regression

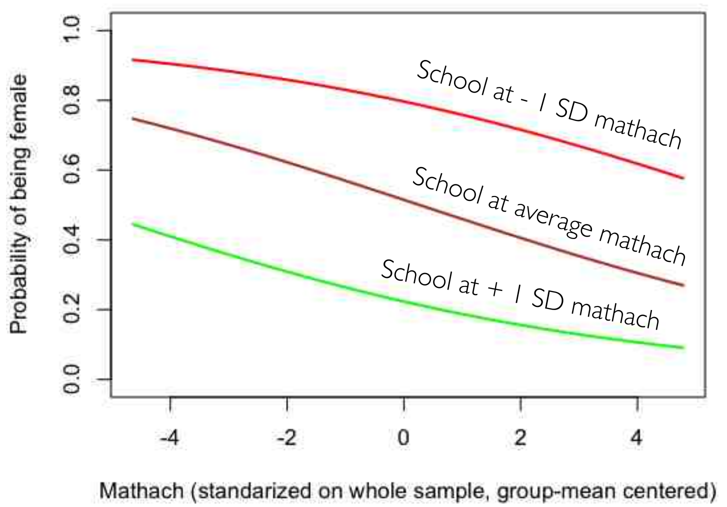



```
# In a first step, mathach has been standardized on the complete
sample

glmer(female ~ mathach.gmc + mathach.gc + (1|id), data=HSB,
family=binomial(link = "logit"))

Random effects:
 Groups Name        Variance Std.Dev.
 id     (Intercept) 4.856    2.204
Number of obs: 7185, groups: id, 160

Fixed effects:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  0.08305    0.18034   0.461  0.64513
mathach.gmc -1.30588    0.40396  -3.233  0.00123 **
mathach.gc  -0.22071    0.03023  -7.301 2.86e-13 ***
```

GLMER + FAMILY SPECIFICATION

WITH GLMM WE GET P VALUES!

```
# plogis() converts a logit into a probability
> plogis(0.08305)
[1] 0.5207506
```



Special models

The Rasch model

```
# Items as fixed (as most IRT programs assume)
glmer(score ~ 0 + item + (1|id), rasch_data, family=binomial)

# Items as random
glmer(score ~ 0 + (1|item) + (1|id), rasch_data, family=binomial)
```

„0 +": OMIT THE INTERCEPT

ONLY 1-PL MODEL POSSIBLE IN LME4, NOT YET THE 2-PL

De Boeck, P., Bakker, M., Zwitser, R., Nivard, M., Hofman, A., Tuerlinckx, F., & Partchev, I. (2011). The estimation of item response models with the lmer function from the lme4 package in R. Journal of Statistical Software, 39(12), 1–28. → http://statmath.wu.ac.at/courses/deboeck/materials/handouts.pdf

Doran, H., Bates, D., Bliese, P., & Dowling, M. (2007). Estimating the multilevel Rasch model: With the lme4 package. Journal of Statistical Software, 20(2), 1–17.

Lamprianou, I. (2013). Application of Single-level and Multi-level Rasch Models using the lme4 Package. *Journal of Applied Measurement, 14*, 79-90.

Interesting email discussion with more information (from 2010): https://stat.ethz.ch/pipermail/r-sig-mixed-models/2010q4/004668.html

Cross-level Mediation

> In all cases, **CWC(M) is recommended** for formulating a multilevel mediation model. [...]
>
> Finally, after analyzing data based on CWC(M), we believe researchers should report results at both levels of analysis, regardless of the level at which the effect should theoretically exist. Reporting both the Level-1 and Level-2 coefficients and mediation effects would facilitate the comparison between levels.
>
> Zhang et al. (2009), p. 715

Recommended Readings

- **General introductions**
- Bolker, B., Brooks, M., Clark, C., Geange, S., Poulsen, J., Stevens, M., & White, J. (2009). Generalized linear mixed models: a practical guide for ecology and evolution. Trends in Ecology & Evolution, 1–28. doi:10.1016/j.tree.2008.10.008
- Gelman, A. (2005). Analysis of variance—why it is more important than ever. The Annals of Statistics, 33, 1–53. doi:10.1214/009053604000001048
- Gelman, A. (2006). Multilevel (Hierarchical) Modeling. Technometrics, 48(3), 432–435. doi:10.1198/004017005000000661
- Hoffman, L., & Rovine, M. J. (2007). Multilevel models for the experimental psychologist: Foundations and illustrative examples. Behavior Research Methods, 39(1), 101–117.
- Kievit, R. A. (2013). Simpson's paradox in psychological science: a practical guide, 1–14. doi:10.3389/fpsyg.2013.00513/abstract
- Quené, H., & den Bergh, van, H. (2004). On multi-level modeling of data from repeated measures designs: A tutorial. Speech Communication, 43(1), 103–122.
- Wright, D. B., & London, K. (2008). Multilevel modelling: Beyond the basic applications. British Journal of Mathematical and Statistical Psychology, 1–18. doi:10.1348/000711008X327632

- **(Cross-level) Interactions**
- Aguinis, H., Gottfredson, R. K., & Culpepper, S. A. 2013. Best-practice recommendations for estimating cross-level interaction effects using multilevel modeling. Journal of Management, 39: 1490-1528.
  Check also the annotated R code, data file, and power calculator described in this article: http://mypage.iu.edu/~haguinis/JOMR.html
- Bauer, D. J., & Curran, P. J. (2005). Probing interactions in fixed and multilevel regression: Inferential and graphical techniques. Multivariate Behavioral Research, 40(3), 373–400.
- Curran, P. J., Bauer, D. J., & Willoughby, M. T. (2004). Testing Main Effects and Interactions in Latent Curve Analysis. Psychological Methods, 9(2), 220–237. doi:10.1037/1082-989X.9.2.220
- Mathieu, J. E., Aguinis, H., Culpepper, S. A., & Chen, G. (2012). Understanding and estimating the power to detect cross-level interaction effects in multilevel modeling. Journal of Applied Psychology, 97: 951-966.
  → http://mypage.iu.edu/~haguinis/crosslevel.html

- **Multilevel Mediation**
- Zhang, Z., Zyphur, M. J., & Preacher, K. J. (2009). Testing Multilevel Mediation Using Hierarchical Linear Models: Problems and Solutions. Organizational Research Methods, 12(4), 695–719. doi:10.1177/1094428108327450xq

- **(Longitudinal) Dyadic Data**
- Bolger, N., & Laurenceau, J.-P. (2013). Intensive longitudinal methods: An introduction to diary and experience sampling research.
- Bolger, N., & Shrout, P. E. (2007). Accounting for statistical dependency in longitudinal data on dyads. In T. D. Little, J. A. Bovaird, & N. A. Card (Eds.), Modeling contextual effects in longitudinal studies. (pp. 285–298). Mahwah, NJ US: Lawrence Erlbaum Associates Publishers.
- Laurenceau, J.-P., & Bolger, N. (2012). Analyzing diary and intensive longitudinal data from dyads. Handbook of research methods for studying daily life, 407–422.
- Raudenbush, S. W., Brennan, R. T., & Barnett, R. C. (1995). A multivariate hierarchical model for studying psychological change within married couples. Journal of Family Psychology, 9(2), 161-174.

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