Practical scale construction: An example

William Revelle

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1 Steps towards scale construction

- 1. Personality scales are not created in a theoretical vacuum. Perhaps the most important step in developing a new scale is a consideration of what is the construct of interest. What is it, what are manifestations of it, what is it not, and what should it not relate to.
- 2. Then, what is the population of interest? Are they young old, highly literate, or somewhat challenged by literacy. Write items suitable for the population of interest.
- 3. Give the items to the participants. Make sure that they are engaged in the task.
- 4. To analyze the data, it is necessary to enter the data into a machine readable form. This is a source of error. Double check for data entry errors. Double entry (two different people enter the data and then the two files are automatically compared) is recommended.
- 5. Run basic descriptive statistics to do one more check for errors.
- 6. Form the variance/covariance matrix from the items and examine the dimensionality of the resulting space.
- 7. Apply various data reduction techniques (factor analysis, principal components analysis, cluster analysis).
- 8. Form composite scales of the selected items. Check these scales for various measures of internal consistency.
- 9. Discriminant validity requires that the scales not correlate with other, unrelated traits.
- 10. Convergent validity requires that the scale do correlate with other, alternative measures of the same trait.

2 A demonstration

As a demonstration of scale construction and validation, consider the following problem. N self report items are given to a number of people. This inventory has is composed of subsets of items that measure believed to measure different traits. In addition, each subject is rated by a friend on those same traits. There are several questions we can ask of these data:

- 1. Do the items form reliable scales?
- 2. What are the correlations of these scales?
- 3. Do the scales correlate with the peer ratings?
- 4. Can we empirically find a better structure of the items?
- 5. Do these revised scales show greater independence, reliability, and validity?

To show the procedures, 14 students in a personality research course spent several weeks learning about each of four personality dimensions. Each student then wrote five items to asses each of four constructs.

- 1. Need for Achievement
- 2. Anxiety
- 3. Sociability
- 4. Impulsivity

As a group they examined all of the items and formed the best 84 items into one questionnaire with 21 items believed to measure each of the constructs. They administered this questionnaire to approximately ten friends each whom they also rated on these four constructs. Thus, we have a data set of about 140 participants assessed on 88 items (the 84 self report items and the 4 peer ratings).

3 Preliminary steps

The data, item labels, and scoring keys are saved on a web server. They may be accessed by the read.table(file.name) command. We then use the dim command to find out the dimensions of the data file as well as the names command to find out what the names are.

```
> prq.data.name <- "http://personality-project.org/revelle/syllabi/301/prq.data"
> prq.keys.name <- "http://personality-project.org/revelle/syllabi/301/prq.keys"
> prq.labels.name <- "http://personality-project.org/revelle/syllabi/301/prq.labels"
> prq.data <- read.table(prq.data.name, header = TRUE)
> prq.keys <- read.table(prq.keys.name, header = TRUE)
> prq.labels <- read.table(prq.labels.name, header = TRUE)
> dim(prq.data)
```

```
[1] 75 91
```

```
> names(prq.data)
```

[1]	"Exp"	"Subject"	"NeedAch"	"Anxiety"	"Sociability"	"Impulsivity"	"G
[8]	"q1"	"q2"	"q3"	"q4"	"q5"	"q6"	"q
[15]	"q8"	"q9"	"q10"	"q11"	"q12"	"q13"	"q
[22]	"q15"	"q16"	"q17"	"q18"	"q19"	"q20"	"q
[29]	"q22"	"q23"	"q24"	"q25"	"q26"	"q27"	"q
[36]	"q29"	"q30"	"q31"	"q32"	"q33"	"q34"	"q
[43]	"q36"	"q37"	"q38"	"q39"	"q40"	"q41"	"q
[50]	"q43"	"q44"	"q45"	"q46"	"q47"	"q48"	"q
[57]	"q50"	"q51"	"q52"	"q53"	"q54"	"q55"	"q
[64]	"q57"	"q58"	"q59"	"q60"	"q61"	"q62"	"q
[71]	"q64"	"q65"	"q66"	"q67"	"q68"	"q69"	"q
[78]	"q71"	"q72"	"q73"	"q74"	"q75"	"q76"	"q
[85]	"q78"	"q79"	"q80"	"q81"	"q82"	"q83"	"q

3.1 Data checking

The first two variables are not particularly interesting, so we create a new data.frame without them. Then find the descriptive statistics of the data in order to make sure that the data were entered correctly.

```
> prq.items <- prq.data[, -c(1:2)]
> describe(prq.items)
```

	var	n	\mathtt{mean}	sd	median	\mathtt{mad}	min	\max	range	skew	kurtosis	se
NeedAch	1	75	6.39	1.92	7	1.48	2	10	8	-0.40	-0.64	0.22
Anxiety	2	75	5.24	2.28	5	2.97	1	10	9	0.09	-1.18	0.26
Sociability	3	75	6.15	2.13	7	1.48	1	9	8	-0.69	-0.60	0.25
Impulsivity	4	75	5.16	2.35	5	2.97	1	9	8	-0.13	-1.32	0.27
Gender	5	74	1.51	0.50	2	0.00	1	2	1	-0.05	-2.02	0.06
q1	6	75	4.27	1.15	4	1.48	1	6	5	-0.52	-0.08	0.13

q2	7 75 3.37 1.39	3 1.48	1	6	5 0.21	-0.73 0.16
q3	8 75 4.36 1.34	5 1.48	1	6	5 -0.57	-0.51 0.15
q4	9 75 4.04 1.33	4 1.48	1	6	5 -0.11	-0.75 0.15
q5	10 75 4.35 1.16	5 1.48	1	6	5 -0.74	0.03 0.13
q6	11 75 3.21 1.41	3 1.48	1	6	5 0.43	-0.85 0.16
q7	12 75 4.17 1.54	5 1.48	1	6	5 -0.44	-0.99 0.18
q8	13 75 3.60 1.49	4 1.48	1	6	5 0.01	-1.11 0.17
q9	14 75 3.96 1.29	4 1.48	1	6	5 -0.38	-0.48 0.15
q10	15 75 3.95 1.51	4 1.48	1	6	5 -0.44	-0.66 0.17
q11	16 75 2.52 1.27	2 1.48	1	6	5 0.62	-0.48 0.15
q12	17 75 3.75 1.50	4 1.48	1	6	5 -0.19	-0.88 0.17
q13	18 75 4.20 1.24	4 1.48	1	6	5 -0.59	-0.03 0.14
q14	19 75 3.07 1.43	3 1.48	1	6	5 0.38	-0.82 0.16
q15	20 75 3.96 1.29	4 1.48	1	6	5 -0.19	-0.98 0.15
q16	21 75 3.71 1.29	4 1.48	1	6	5 -0.31	-0.49 0.15
q17	22 75 4.40 1.21	4 1.48	1	6	5 -0.52	-0.27 0.14
q18	23 75 2.80 1.34	3 1.48	1	6	5 0.43	-0.80 0.15
q19	24 75 4.21 1.30	4 1.48	1	6	5 -0.36	-0.46 0.15
q20	25 75 3.77 1.28	4 1.48	1	6	5 0.19	-0.88 0.15
q21	26 75 3.81 1.45	4 1.48	1	6	5 -0.23	-0.89 0.17
q22	27 75 4.61 1.22	5 1.48	1	6	5 -0.70	-0.06 0.14
q23	28 75 4.15 1.28	4 1.48	1	6	5 -0.31	-0.72 0.15
q24	29 75 3.53 1.23	4 1.48	1	6	5 0.07	-0.81 0.14
q25	30 75 4.03 1.03	4 1.48	2	6	4 -0.13	-0.76 0.12
q26	31 75 3.87 1.38	4 1.48	1	6	5 -0.10	-1.02 0.16
q27	32 75 4.72 1.17	5 1.48	1	6	5 -1.01	1.17 0.13
q28	33 75 2.88 1.44	3 1.48	1	6	5 0.18	-1.06 0.17
q29	34 75 4.45 1.45	5 1.48	1	6	5 -0.60	-0.78 0.17
q30	35 75 3.24 1.36	3 1.48	1	6	5 0.07	-0.93 0.16
q31	36 75 3.23 1.52	3 1.48	1	6	5 0.35	-1.05 0.18
q32	37 75 4.07 1.23	4 1.48	1	6	5 -0.42	-0.14 0.14
q33	38 75 4.04 1.32	4 1.48	1	6	5 -0.73	-0.29 0.15
q34	39 75 3.92 1.38	4 1.48	1	6	5 -0.13	-0.74 0.16
q35	40 75 4.01 1.39	4 1.48	1	6	5 -0.14	-1.03 0.16
q36	41 75 3.40 1.32	4 1.48	1	6	5 0.02	-0.82 0.15
q37	42 75 4.13 1.14	4 1.48	1	6	5 -0.26	-0.32 0.13
q38	43 75 3.37 1.46	3 1.48	1	6	5 0.20	-1.02 0.17
q39	44 75 3.87 1.40	4 1.48	1	6	5 0.12	-1.16 0.16
q40	45 75 3.91 1.19	4 1.48	1	6	5 -0.01	-0.57 0.14
q41	46 75 4.39 1.17	5 1.48	1	6	5 -0.47	-0.37 0.14
q42	47 75 3.33 1.51	3 1.48	1	6	5 0.04	-1.21 0.17

q43	48 75 4.00	1.22	4	1.48	1	6	5	-0.44	-0.16	6 0.14
q44	49 75 3.33	1.42	3	1.48	1	6	5	0.05	-0.92	2 0.16
q45	50 75 4.25	1.15	4	1.48	2	6	4	-0.50	-0.44	0.13
q46	51 75 2.93	1.44	3	1.48	1	6	5	0.47	-0.83	3 0.17
q47	52 75 3.91	1.42	4	1.48	1	6	5	-0.34	-0.79	0.16
q48	53 75 4.20	1.08	4	1.48	2	6	4	-0.20	-0.63	0.12
q49	54 75 4.57	1.23	5	1.48	1	6	5	-0.91	0.53	0.14
q50	55 75 3.28	1.55	3	1.48	1	6	5	0.25	-1.13	8 0.18
q51	56 75 3.24	1.49	3	1.48	1	6	5	0.37	-0.84	0.17
q52	57 75 3.83	1.42	4	1.48	1	6	5	-0.03	-1.10	0.16
q53	58 75 3.40	1.25	3	1.48	1	6	5	0.16	-0.59	0.14
q54	59 75 3.67	1.61	4	1.48	1	6	5	-0.13	-1.10	0.19
q55	60 75 4.44	1.30	5	1.48	1	6	5	-0.51	-0.59	0.15
q56	61 75 3.92	1.51	4	1.48	1	6	5	-0.30	-1.03	3 0.17
q57	62 75 3.97	1.17	4	1.48	1	6	5	-0.15	-0.61	0.14
q58	63 75 4.56	1.23	5	1.48	1	6	5	-0.76	-0.10	0.14
q59	64 75 2.85	1.50	3	1.48	1	6	5	0.55	-0.74	0.17
q60	65 75 3.60	1.40	4	1.48	1	6	5	0.06	-0.95	5 0.16
q61	66 75 4.91	1.04	5	1.48	1	6	5	-1.16	1.93	8 0.12
q62	67 75 3.41	1.58	3	1.48	1	6	5	0.27	-1.13	8 0.18
q63	68 75 3.17	1.55	3	1.48	1	6	5	0.18	-1.04	0.18
q64	69 75 3.59	1.49	4	1.48	1	6	5	-0.08	-0.96	6 0.17
q65	70 75 3.60	1.30	4	1.48	1	6	5	-0.11	-0.75	5 0.15
q66	71 75 3.73	1.43	4	1.48	1	6	5	-0.19	-1.00	0.16
q67	72 75 3.72	1.56	4	1.48	1	6	5	-0.15	-1.05	5 0.18
q68	73 75 3.61	1.31	4	1.48	1	6	5	-0.05	-0.70	0.15
q69	74 75 3.61	1.50	3	1.48	1	6	5	0.07	-1.10	0.17
q70	75 75 3.85	1.20	4	1.48	1	6	5	-0.27	-0.55	5 0.14
q71	76 75 3.40	1.62	3	1.48	1	6	5	0.10	-1.25	5 0.19
q72	77 75 3.40	1.16	4	1.48	1	6	5	-0.19	-0.35	5 0.13
q73	78 75 4.03	1.24	4	1.48	1	6	5	-0.34	-0.51	0.14
q74	79 75 3.80	1.15	4	1.48	1	6	5	-0.24	-0.76	0.13
q75	80 75 3.25	1.38	3	1.48	1	6	5	0.16	-0.91	0.16
q76	81 75 3.45	1.52	4	1.48	1	6	5	-0.05	-1.08	8 0.18
q77	82 75 3.93	1.20	4	1.48	1	6	5	-0.47	-0.20	0.14
q78	83 75 3.04	1.29	3	1.48	1	6	5	0.60	-0.40	0.15
q79	84 75 4.03	1.56	4	1.48	1	6	5	-0.34	-1.06	6 0.18
q80	85 75 3.91	1.30	4	1.48	1	6	5	-0.16	-0.73	8 0.15
q81	86 75 4.32	1.22	4	1.48	1	6	5	-0.57	0.13	8 0.14
q82	87 75 3.84	1.46	4	1.48	1	6	5	-0.39	-0.74	0.17
q83	88 75 4.08	1.33	4	1.48	2	6	4	-0.35	-1.06	6 0.15

3.2 Data revision

In doing this, we discovered (on the first pass through the data) that one of the variables had a range of 32 rather than the 6 that was appropriate. Correcting the data, we can start over again. Even with well meaning, careful data entry, mistakes will happen in data entry. It is recommended that data be entered twice and then compared using software that compares the two files line by line and entry by entry. In all cases, make sure to describe the data and check that the ranges are appropriate for the data.

Thus, the data were edited and the prior steps were done again until there were no incorrectly entered subjects. One error that makes data checking complicated is a blank field in Excel is read improperly. Using NA to specify not available is better. Note that the describe output shows that some variables do not have as many subjects as others.

4 Score the scales

Forming scale scores as linear sums (or averages) of the items is easy to do in R. One technique (not recommended) is to a series of recodings, creating new variables for each scale. A simpler technique, using the score items function from the psych package does this for all scales defined in a matrix of keys (the keys matrix. This is essentially a matrix of -1, 0, and 1s where 0 means don't include the item in the scale, and a 1 means to include it. -1 means to reverse key the item.

We first do this just for the items. The item.scores is a list of multiple values:

- 1. scores the actual scores for each subject
- 2. missing where there any missing values for any subject?
- 3. alpha coefficient alpha for each scale
- 4. av.r the average r within each scale
- 5. n.items how many items in each scale?
- 6. item.cor the correlation of each item with each scale
- 7. cor the correlation matrix of the scales
- 8. corrected the raw correlations of the scales (below the diagonal), the alpha reliabilities (on the diagonal), and the intercorrelations corrected for unreliability (above the diagonal).

6

Using different elements of this list, we can do different things. For this early analysis, we just show the values in "corrected". Note the use of the \$ to show that we want a particular list element.

```
> item.scores <- score.items(prq.keys[, 6:9], prq.items)</pre>
> print(item.scores$corrected)
      Nach
             Anx
                    Soc
                          Imp
      0.84
            0.08
                  0.28 -0.23
Nach
Anx
      0.07
            0.82 - 0.25
                         0.09
Soc
      0.24 -0.22 0.89
                         0.44
    -0.19 0.08 0.39
                         0.87
Imp
```

5 Discriminant Validity of the scales and of the peer ratings

The self report scales were meant to assess four different constructs. If this is the case, then we would expect the pattern of correlations between them to be low.

5.1 Discrimant validity of self report

We have already seen this from the output of the score items function (see above). We can show this graphically (Figure 1).

5.2 Discriminant validity of the peer ratings

We can also show the peer rating correlations graphically to examine the discrimant validity of the peer ratings (Figure 2).

6 Convergent validity

Both Figure 1 and Figure 2 suggest that the constructs are fairly independent. But does self report match peer ratings? We can examine this by finding the correlation matrix of self ratings with peer ratings:

> round(cor(prq.data[, 3:7], item.scores\$scores, use = "pairwise"), 2)

	Nach	Anx	Soc	Imp
NeedAch	0.18	0.09	0.00	-0.29
Anxiety	-0.01	0.60	-0.21	0.05
Sociability	-0.14	-0.16	0.57	0.35
Impulsivity	-0.23	0.15	0.21	0.50
Gender	-0.09	0.21	-0.01	0.07

> pairs.panels(item.scores\$scores)



Figure 1: A scatter plot matrix (SPLOM) of the self report scores shows that the four scales are relatively independent.

> pairs.panels(prq.data[, 3:7])



Figure 2: A scatter plot matrix (SPLOM) of the peer ratings also shows that the traits are rated as independent.

It seems that three of the traits show high convergent validity, but one (Need for Achievement) doe not.

7 Construct validity: Convergent and Discriminant

Do measures of the same construct correlate across methods of assessment? Do measures of different constructs not correlate within or across methods of assessment? These questions can be addressed by simultaneously examining the correlations within peer ratings, within self report, and between the self report and peer ratings. This is the well known "multi-trait-multi-method correlation matrix.

We can find this matrix by combining the two sets of analyses into one analysis. We do this by keying the first five items (the peer ratings) along with the next 84 items (the self reports) and using the score.items function.

```
> item.scores <- score.items(prq.keys, prq.items)
> print(item.scores$corrected)
```

	PNach	PAnx	PSoc	PImp	G	Nach	Anx	Soc	Imp
PNach	1.00	0.21	-0.08	-0.30	0.02	0.20	0.10	0.00	-0.31
PAnx	0.21	1.00	-0.10	-0.03	0.38	-0.01	0.66	-0.22	0.06
PSoc	-0.08	-0.10	1.00	0.29	0.08	-0.16	-0.18	0.60	0.37
PImp	-0.30	-0.03	0.29	1.00	0.06	-0.25	0.16	0.22	0.53
G	0.02	0.38	0.08	0.06	1.00	-0.10	0.23	-0.01	0.07
Nach	0.18	-0.01	-0.14	-0.23	-0.09	0.84	0.08	0.28	-0.23
Anx	0.09	0.60	-0.16	0.15	0.20	0.07	0.82	-0.25	0.09
Soc	0.00	-0.21	0.57	0.21	-0.01	0.24	-0.22	0.89	0.44
Imp	-0.29	0.05	0.35	0.50	0.07	-0.19	0.08	0.39	0.87

This can also be shown graphically (Figure 3).

8 Determining how many constructs are in a set of items

The items analysed were meant to represent four constructs. Given the previous analysis, they probably do. But what if we did not know how many separate dimensions were in the data? Is it possible to find out? Three alternative procedure address this question.

- 1. Principal componenents analysis
- 2. Factor analysis
- 3. Cluster analysis



> pairs.panels(item.scores\$scores[, -5])

Figure 3: A scatter plot matrix (SPLOM) of multitraits-multimethods matrix.

All three of these procedures are attempting to approximate the nvar * nvar correlation matrix R with a matrix of lesser rank, one that is nvar * nf. That is, can we find a Factor (Component or Cluster) such that

$$R \approx FF' \tag{1}$$

8.1 Principal Components Analysis: PCA

A standard matrix algrebra procedure is the eigen value decomposition of a matrix. The eigen values are scalers which when multiplied by a matrix of (orthogoanl) eigen vectors, V, are the roots of a matrix equation. We can represent a square matrix, R, by the product of an orthogonal matrix, V times a diagonal matrix, D, times the transpose of V:

$$R = VDV' \tag{2}$$

The principal components, C, of a matrix are simply $C = V * \operatorname{sqrt}(D)$ and thus

$$R = CC' \tag{3}$$

1

0

0

0

Although equation 3 can reproduce the matrix, if we take just the first n components, we can approximate the correlation pretty well. As $n \rightarrow nvar$ the approximation beccomes better.

To do this in R, we use the principal function. Although the principal function will find the correlations for us, it is useful to calculate the correlations first, so that we can then add in variable labels.

> r.prq <- cor(prq.items, use = "pairwise")
> rownames(r.prq) <- colnames(r.prq) <- strtrim(prq.labels[, 2], 40)</pre>

8.2 How many variables to extract?

There are a number of tests that have been developed to determine the optimal number of factors or components to remove to summarize a matrix. One of the most straight forward is Cattell's "scree test".

```
> pc <- principal(r.prq, 4, rotate = TRUE)
> print(pc)
$values
 [1] 12.44
             9.35
                   7.40
                          4.91
                                3.79
                                       3.55
                                             3.23
                                                    3.02
                                                          2.66
                                                                 2.61
                                                                        2.22
                                                                              2.15
                                                                                     1.95
                                                                                           1.85
[17]
      1.45
             1.38
                   1.32
                          1.22
                                1.17
                                       1.13
                                             1.07
                                                    1.02
                                                          0.93
                                                                 0.90
                                                                        0.83
                                                                              0.79
                                                                                     0.74
                                                                                           0.70
[33]
                          0.50
                                0.46
                                       0.44
                                             0.43
                                                    0.39
                                                          0.37
                                                                 0.33
                                                                                     0.28
      0.61
             0.56
                   0.55
                                                                        0.32
                                                                              0.30
                                                                                           0.28
[49]
                                                                                     0.08
      0.23
            0.21
                   0.19
                          0.18
                                0.17
                                       0.17
                                             0.14
                                                    0.13
                                                          0.12
                                                                 0.12
                                                                        0.10
                                                                              0.09
                                                                                           0.07
[65]
      0.06
            0.05
                   0.05
                          0.03
                                0.03
                                       0.02
                                             0.02
                                                    0.01
                                                          0.01
                                                                 0.00
                                                                       0.00
                                                                              0.00
                                                                                     0.00
                                                                                           0.00
```

> VSS.scree(r.prq)

.



Figure 4: The scree test may be used to estimate the most interpretable number of factors or components

> VSS.plot(VSS(r.prq, 8, pc = "pc", rotate = "varimax"))



.

Very Simple Structure

Figure 5: The Very Simple Structure test is an alternative way for determining the optimal number of factors/components

\$loadings

Loadings:

	PC1	PC2	PC3	PC4
NeedAch		0.22	0.10	0.27
Anxiety	-0.24		0.59	-0.18
Sociability	0.63	-0.26		-0.12
Impulsivity	0.27	-0.29	0.14	-0.43
Gender			0.20	-0.15
I love to seek out new challenges	0.27	0.58	-0.14	
I get nervous very easily	-0.22	-0.22	0.62	
I like to meet new people in everyday s	0.75	0.17	0.10	0.10
I am thoughtful and deliberate when mak		0.60	-0.12	0.38
Personal satisfaction is the best rewar		0.42	0.23	-0.13
I dont handle stress well	-0.19	-0.15	0.71	
I can easily start conversations with p	0.54	0.15		
I say things that I regret later	0.12	-0.15	0.10	-0.62
I am a good multi tasker	0.30	0.30		0.21
I am easily bothered by negative reviews			0.60	
I tend to avoid social situations	-0.77	-0.15	0.13	
I weigh all the options carefully befor	-0.17	0.40		0.25
I like to go the extra mile on a projec	0.13	0.65		0.12
Measures of skill or intelligence make m	-0.23		0.30	-0.10
I tend to lead the conversation	0.42		-0.28	
I tend to make decisions quickly	0.39	0.18	-0.22	-0.51
I have high standards for the quality o	0.13	0.67	0.14	0.18
I rarely feel tense	0.26	-0.24	-0.48	0.24
I am good at maintaining a lively conve	0.56	0.29	-0.13	-0.19
I plan my activities in advance		0.33	0.16	0.47
I am a perfectionist	-0.12	0.33	0.29	0.13
I feel stressed when I have a lot to do		0.22	0.58	0.21
I make friends easily	0.72	0.20	-0.13	-0.16
I often change my plans at the last min	0.18			-0.72
If I fail, I keep trying until I succee	0.29	0.58		
I often feel anxious about future events	-0.14	0.22	0.54	-0.22
I tend to enjoy small groups of people	-0.27	0.44		-0.11
I dislike planning ahead		-0.17	0.12	-0.58
I seek the enjoyment of winning	0.18	0.27	-0.14	-0.13
I often feel tense, nauseous, and/or fai	-0.13		0.49	-0.23

I tend to talk a lot in large groups	0.40	0.19	-0.45	-0.30
I indulge in my desires on a whim	0.19	0.15	0.13	-0.54
I find myself needing to achieve whatev	0.20	0.66		
I have a hard time forgetting negative e	-0.13	0.21	0.56	
I have a large social network	0.80		-0.17	-0.14
When working on a necessary task and a	-0.13	0.22		0.25
I get bored if a task is not challengin	0.13	0.40	-0.14	-0.21
I often have unwanted and/or disturbing	-0.31		0.22	-0.56
Id rather spend time with others than s	0.72			
I act on sudden urges				-0.72
I always make sure anything attached to	0.12	0.62		
Even trivial problems greatly contribut	-0.21		0.68	
I am happier when Im around other peopl	0.63	0.37		-0.20
I often regret decisions because I acte	0.21	-0.13	0.29	-0.56
I prefer challenging tasks to easy ones	0.12	0.49	-0.11	
I often have difficulty sleeping	-0.28	0.15		-0.37
I enjoy being alone	-0.48		0.11	0.18
I tend to act on my gut feelings	0.11	0.29	-0.17	-0.48
The joy of success is worth the hard wo	0.22	0.58		
Even in non stressful situations, I fin			0.66	
People are more likely to initiate a co	-0.63	-0.15	0.16	0.20
I often get sidetracked in the middle o		-0.14	0.15	-0.69
I only work as hard as I have to on tas	0.11	-0.28	0.18	-0.22
I feel tension in my body or face while	-0.24	0.18	0.27	-0.29
Ill spend time talking to a friend even	0.33	-0.13	0.37	-0.32
I often and actively express my feeling	0.55		0.21	-0.22
I always reach the goals I set for myse	0.25	0.46		0.14
I prefer to work in relaxed environment		0.45	0.11	-0.19
I prefer large crowded parties to small	0.55	-0.14	-0.13	-0.12
I stay on task until a project is compl		0.61		
I experience great joy when my efforts		0.56		
A small unpleasant event can ruin my da		0.17	0.60	
A good night for me is reading a book	-0.42	-0.11	0.29	0.10
I dislike changing established plans			0.43	0.43
I tend to have trouble getting motivate		-0.43	0.32	-0.12
I worry about what others think of me	0.15	-0.10	0.62	
I am always willing to attend a party	0.57	0.14	-0.10	-0.32
I always think before I act	-0.14	0.24		0.53
I tend to procrastinate and waste more	0.28	-0.43	0.12	-0.31
I bounce back quickly from unpleasant s	0.29	0.34	-0.43	-0.18
I dont understand how people can spend	0.43	-0.24	-0.15	

I always stick to plans	0.22		0.19	0.51
I set long term and sizeable goals for	-0.15	0.55		-0.12
I tend to dwell on obstacles in the nea	0.15	0.28	0.44	-0.13
I work better when there are people aro	0.15	-0.22		
I sometimes look back and dont know why	0.23	0.10		-0.52
I always see projects through to the fi		0.61	0.13	0.10
I tend to back away from tasks I think	0.14	-0.52	0.36	
When given the choice, I will work alon	-0.48	0.31		
I often say the first thing that comes	0.33	-0.18		-0.40
I believe that if something is worth doi	0.16	0.76		-0.12
I am more emotional than my friends	0.28		0.46	
I am a very sociable person	0.77	0.32	-0.16	-0.11
I am an impulsive person	0.27			-0.61
PC1 PC2 PC3 PC4				

SS loadings 9.77 9.399 7.379 7.584 Proportion Var 0.11 0.106 0.083 0.085 Cumulative Var 0.11 0.215 0.298 0.384

\$fit [1] 0.76

8.3 Factor Analyis:FA

The factor analysis model is a generalization of principal components, but one that estimates just the shared variance between variables rather the all of the variance of variable. It is done by repetively finding the eigen values and vectors of a matrix where the diagonal of the matrix is estimated from the previous interation. When the diagonals no longer change, the principal factors solution is said to have converged.

> fa4 <- factor.pa(r.prq, 4)
> print(fa4)

\$values

```
[1] 11.89
                 6.78
                       4.30
                              3.09
                                   2.86
                                         2.57
                                                2.35
                                                     1.99
                                                            1.89
                                                                  1.56
                                                                        1.45
                                                                              1.33
           8.71
                                                                                    1.17
                                                                                         1
[17] 0.79
                 0.66 0.57
                                               0.35 0.28 0.24
                                                                  0.20
                                                                       0.14
                                                                             0.11
           0.72
                             0.50
                                   0.47
                                         0.41
                                                                                   0.04
                                                                                          0
[33] -0.03 -0.06 -0.11 -0.14 -0.16 -0.17 -0.19 -0.22 -0.23 -0.27 -0.30 -0.31 -0.33 -0.34 -0
[49] -0.38 -0.39 -0.40 -0.43 -0.43 -0.44 -0.46 -0.47 -0.49 -0.49 -0.50 -0.51 -0.53 -0.53 -0
[65] -0.57 -0.58 -0.58 -0.59 -0.59 -0.60 -0.60 -0.62 -0.62 -0.62 -0.64 -0.65 -0.65 -0.66 -0
[81] -0.68 -0.69 -0.70 -0.71 -0.72 -0.73 -0.75 -0.78 -0.80
```

\$loadings

Loadings:

8	D 4 4	D 4 O	D 4 0	544
	PA1	PA2	PA3	PA4
NeedAch		0.21		0.25
Anxiety	-0.23		0.57	-0.16
Sociability	0.61	-0.26		-0.12
Impulsivity	0.26	-0.28	0.13	-0.41
Gender			0.19	-0.13
I love to seek out new challenges	0.26	0.57	-0.14	
I get nervous very easily	-0.22	-0.22	0.61	
I like to meet new people in everyday s	0.74	0.16		0.10
I am thoughtful and deliberate when mak		0.59	-0.12	0.37
Personal satisfaction is the best rewar		0.39	0.22	-0.11
I dont handle stress well	-0.19	-0.15	0.69	
I can easily start conversations with p	0.51	0.15		
I say things that I regret later	0.12	-0.14	0.10	-0.59
I am a good multi tasker	0.28	0.29		0.18
I am easily bothered by negative reviews			0.57	
I tend to avoid social situations	-0.77	-0.15	0.14	
I weigh all the options carefully befor	-0.16	0.38		0.24
I like to go the extra mile on a projec	0.13	0.63		0.12
Measures of skill or intelligence make m	-0.22		0.28	
I tend to lead the conversation	0.40		-0.27	
I tend to make decisions quickly	0.39	0.18	-0.21	-0.49
I have high standards for the quality o	0.13	0.66	0.13	0.17
I rarely feel tense	0.24	-0.23	-0.47	0.22
I am good at maintaining a lively conve	0.55	0.28	-0.13	-0.19
I plan my activities in advance		0.32	0.15	0.45
I am a perfectionist	-0.12	0.31	0.27	0.12
I feel stressed when I have a lot to do		0.21	0.55	0.21
I make friends easilv	0.71	0.20	-0.13	-0.16
I often change my plans at the last min	0.18			-0.71
If I fail. I keep trying until I succee	0.28	0.56		
I often feel anxious about future events	-0.14	0.21	0.51	-0.21
I tend to enjoy small groups of people	-0.24	0.41	0.01	
I dislike planning ahead	0.21	-0 17	0 12	-0 55
I seek the enjoyment of winning	0 17	0.25	-0 13	-0.13
I often feel tense nauseous and/or fai	-0 12	0.20	0.10	-0.21
I tend to talk a lot in large groups	0.12	0 10	-0 /3	-0.30
I indulge in my desires on a whim	0.09	0.14	0.40	-0 52
I find musclf needing to achieve whetew	0.19	0.14	0.12	0.02
T TIM WASELT HEEMTHE TO SCHIEVE MUSICAL	0.19	0.05		

I have a hard time forgetting negative e	-0.13	0.20	0.53	
I have a large social network	0.80		-0.17	-0.14
When working on a necessary task and a	-0.12	0.21		0.23
I get bored if a task is not challengin	0.14	0.37	-0.13	-0.19
I often have unwanted and/or disturbing	-0.29		0.22	-0.54
Id rather spend time with others than s	0.70			
I act on sudden urges				-0.70
I always make sure anything attached to	0.12	0.60		
Even trivial problems greatly contribut	-0.21		0.66	
I am happier when Im around other peopl	0.62	0.36		-0.19
I often regret decisions because I acte	0.20	-0.13	0.28	-0.54
I prefer challenging tasks to easy ones	0.12	0.47	-0.11	
I often have difficulty sleeping	-0.26	0.13		-0.33
I enjoy being alone	-0.46		0.11	0.18
I tend to act on my gut feelings	0.11	0.28	-0.16	-0.45
The joy of success is worth the hard wo	0.22	0.56		
Even in non stressful situations, I fin			0.63	
People are more likely to initiate a co	-0.61	-0.15	0.16	0.20
I often get sidetracked in the middle \ensuremath{o}		-0.14	0.15	-0.67
I only work as hard as I have to on tas	0.10	-0.27	0.17	-0.21
I feel tension in my body or face while $% \left({{{\mathbf{x}}_{i}}} \right)$	-0.22	0.17	0.27	-0.26
Ill spend time talking to a friend even	0.32	-0.13	0.35	-0.30
I often and actively express my feeling $% \left[{{\left[{{\left[{{\left[{\left[{\left[{\left[{\left[{\left[{$	0.53		0.19	-0.22
I always reach the goals I set for myse	0.23	0.44		0.13
I prefer to work in relaxed environment		0.43	0.10	-0.18
I prefer large crowded parties to small	0.52	-0.14	-0.13	-0.12
I stay on task until a project is compl		0.59		
I experience great joy when my efforts		0.54		
A small unpleasant event can ruin my da		0.17	0.57	
A good night for me is reading a book	-0.40	-0.10	0.28	0.10
I dislike changing established plans			0.40	0.41
I tend to have trouble getting motivate $% \left[{{\left[{{\left[{{\left[{\left[{\left[{\left[{\left[{\left[{$		-0.41	0.30	-0.12
I worry about what others think of me	0.14		0.59	
I am always willing to attend a party	0.55	0.13	-0.10	-0.31
I always think before I act	-0.13	0.23		0.51
I tend to procrastinate and waste more	0.27	-0.41	0.12	-0.30
I bounce back quickly from unpleasant s	0.29	0.33	-0.41	-0.17
I dont understand how people can spend	0.40	-0.23	-0.15	
I always stick to plans	0.19		0.17	0.47
I set long term and sizeable goals for	-0.13	0.52		-0.11
I tend to dwell on obstacles in the nea	0.14	0.27	0.42	-0.12

I work better when there are people aro 0.13 - 0.20I sometimes look back and dont know why 0.23 -0.49I always see projects through to the fi 0.59 0.12 I tend to back away from tasks I think 0.13 -0.51 0.35 When given the choice, I will work alon -0.44 0.29 I often say the first thing that comes 0.31 -0.17 -0.39 I believe that if something is worth doi 0.16 0.75 -0.11 I am more emotional than my friends 0.25 0.42 I am a very sociable person 0.77 0.31 -0.16 -0.10 I am an impulsive person 0.27 -0.59PA1 PA2 PA3 PA4 SS loadings 9.207 8.777 6.762 6.945 Proportion Var 0.103 0.099 0.076 0.078 Cumulative Var 0.103 0.202 0.278 0.356 \$fit [1] 0.75 \$fitoff [1] 0.8 \$communality [1] 34.10 31.80 31.69 31.69 31.68

8.4 Cluster analysis of items: ICLUST

Yet another alternative is a very simple procedure of clustering the most similar variables together and repeating this process until the cluster reliabilities do not increase by combining the clusters.

> ic <- ICLUST(prq.items, labels = strtrim(prq.labels[, 2], 20))</pre> > print(ic\$cor) C84 C81 C82 C79 C80 C84 1.00 0.21 0.04 0.11 -0.31 C81 0.21 1.00 0.03 -0.19 -0.05 C82 0.04 0.03 1.00 -0.22 0.25 C79 0.11 -0.19 -0.22 1.00 0.36 C80 -0.31 -0.05 0.25 0.36 1.00 > print(ic\$alpha)

C84 C81 C82 C79 C80 0.86 0.57 0.89 0.87 0.92

> print(ic\$sorted)

\$sorted

	item	content	cluster	C84	C81	C82	C79	C80
q42	47	Even trivial proble	1	0.65	0.12	-0.07	-0.04	-0.28
Anxiety	2	Anxiety	1	0.62	-0.02	-0.04	0.03	-0.23
q6	11	I dont handle stress	1	0.61	0.21	-0.15	0.03	-0.34
q50	55	Even in non stressf	1	0.58	0.34	-0.02	0.01	-0.15
q2	7	I get nervous very e	1	0.55	0.12	-0.19	0.00	-0.34
q18	23	I rarely feel tense	1	-0.52	0.02	-0.16	-0.06	0.19
q34	39	I have a hard time ${\tt f}$	1	0.52	0.26	0.23	-0.09	-0.17
q10	15	I am easily bothered	1	0.50	0.16	-0.02	0.07	-0.15
q26	31	I often feel anxious	1	0.49	0.18	0.22	0.16	-0.16
q22	27	I feel stressed when	1	0.48	0.23	0.23	-0.17	-0.19
q30	35	I often feel tense,	1	0.48	0.09	-0.05	0.20	-0.19
q62	67	A small unpleasant	1	0.47	0.24	0.17	-0.05	-0.14
q66	71	I worry about what	1	0.45	0.22	-0.02	0.12	-0.02
q38	43	I often have unwant	1	0.38	-0.16	-0.02	0.32	-0.16
q54	59	I feel tension in m	1	0.38	-0.26	0.12	0.10	-0.13
q74	79	I tend to dwell on	1	0.38	0.05	0.25	0.12	0.11
q14	19	Measures of skill or	1	0.34	-0.18	-0.12	0.01	-0.22
q21	26	I am a perfectionis	1	0.29	0.18	0.28	-0.21	-0.07
Gender	5	Gender	1	0.23	-0.13	-0.06	0.05	-0.04
q46	51	I often have diffic	1	0.17	-0.12	0.06	0.12	-0.15
q72	77	I always stick to p	2	0.01	0.44	0.14	-0.30	0.05
q64	69	I dislike changing	2	0.20	0.40	0.01	-0.20	-0.19
q82	87	I am more emotional	2	0.27	0.33	-0.07	0.13	0.09
q81	86	I believe that if so	3	0.06	0.02	0.69	-0.02	0.27
q17	22	I have high standar	3	0.10	0.21	0.66	-0.21	0.11
q33	38	I find myself needi	3	0.06	0.24	0.64	0.01	0.26
q4	9	I am thoughtful and	3	-0.11	0.08	0.58	-0.43	0.10
q41	46	I always make sure	3	-0.01	0.09	0.58	-0.14	0.19
q13	18	I like to go the ex	3	0.05	0.14	0.56	-0.23	0.18
q25	30	If I fail, I keep t	3	-0.11	0.26	0.56	-0.05	0.30
q1	6	I love to seek out	3	-0.07	0.05	0.54	-0.02	0.39
q61	66	I experience great	3	-0.05	0.03	0.54	-0.17	0.15
q77	82	I always see projec	3	0.15	0.18	0.54	-0.17	0.16
q49	54	The joy of success	3	0.01	0.08	0.53	0.00	0.26

q60	65 I stay on task unti	3 0.11	0.15	0.52	-0.24	0.15
q45	50 I prefer challengin	3 -0.11	0.19	0.48	-0.05	0.12
q78	83 I tend to back away	3 0.20	0.08	-0.47	0.22	-0.03
q73	78 I set long term and	3 0.11	-0.06	0.45	-0.12	0.03
q12	17 I weigh all the opt	3 0.04	-0.05	0.43	-0.28	-0.16
q57	62 I always reach the	3 -0.10	0.22	0.43	-0.13	0.26
q37	42 I get bored if a ta	3 -0.08	-0.12	0.39	0.11	0.21
q58	63 I prefer to work in	3 0.16	-0.07	0.39	0.08	0.09
q5	10 Personal satisfacti	3 0.21	0.07	0.35	-0.01	-0.03
q27	32 I tend to enjoy sma	3 0.17	-0.05	0.34	-0.09	-0.19
q9	14 I am a good multi t	3 -0.12	0.22	0.31	-0.12	0.25
q29	34 I seek the enjoymen	3 -0.15	0.00	0.26	0.12	0.25
q75	80 I work better when	3 -0.03	-0.01	-0.20	0.11	0.17
q24	29 I often change my p	4 0.08	-0.30	0.07	0.63	0.28
q52	57 I often get sidetra	4 0.21	-0.34	-0.16	0.61	0.13
q8	13 I say things that I	4 0.12	-0.20	-0.14	0.57	0.19
q40	45 I act on sudden urg	4 0.03	-0.29	0.06	0.56	0.17
q28	33 I dislike planning	4 0.12	-0.17	-0.16	0.55	0.03
q84	89 I am an impulsive pe	4 -0.07	-0.12	0.04	0.55	0.33
q44	49 I often regret deci	4 0.28	-0.20	-0.14	0.54	0.26
q69	74 I tend to procrasti	4 -0.04	-0.08	-0.31	0.52	0.18
Impulsivity	4 Impulsivity	4 0.08	0.03	-0.25	0.50	0.23
q32	37 I indulge in my des	4 0.13	0.09	0.13	0.48	0.22
q76	81 I sometimes look ba	4 0.10	-0.09	0.06	0.46	0.31
q80	85 I often say the fir	4 -0.14	0.02	-0.15	0.46	0.35
q20	25 I plan my activitie	4 0.15	0.24	0.29	-0.45	-0.09
q55	60 Ill spend time talk	4 0.21	-0.04	-0.07	0.45	0.22
q68	73 I always think befo	4 -0.03	0.15	0.29	-0.43	-0.22
q53	58 I only work as hard	4 0.09	-0.15	-0.19	0.35	0.05
q65	70 I tend to have trou	4 0.19	0.07	-0.32	0.33	-0.09
NeedAch	1 NeedAch	4 0.15	0.02	0.17	-0.32	-0.03
q36	41 When working on a n	4 -0.01	0.20	0.14	-0.31	-0.11
q83	88 I am a very sociable	5 -0.28	0.05	0.37	0.25	0.78
q35	40 I have a large soci	5 -0.31	0.05	0.18	0.39	0.77
q11	16 I tend to avoid soc	5 0.32	-0.20	-0.21	-0.30	-0.68
q23	28 I make friends easi	5 -0.28	0.11	0.26	0.36	0.66
q51	56 People are more lik	5 0.20	0.09	-0.19	-0.32	-0.66
q3	8 I like to meet new	5 -0.12	0.28	0.25	0.19	0.63
q39	44 Id rather spend tim	5 -0.06	0.20	0.11	0.30	0.63
q43	48 I am happier when I	5 -0.08	0.15	0.43	0.34	0.62
q67	72 I am always willing	5 -0.12	-0.11	0.17	0.40	0.62

q19	24	I am good at mainta	5 -0.1	5 0.07	0.25	0.21	0.61
q31	36	I tend to talk a lo	5 -0.34	⊢ 0.26	0.15	0.23	0.56
Sociability	3	Sociability	5 -0.20	0.03	-0.18	0.37	0.54
q59	64	I prefer large crow	5 -0.18	0.02	-0.10	0.28	0.54
q16	21	I tend to make deci	5 -0.10	5 -0.05	0.15	0.44	0.51
q47	52	I enjoy being alone	5 0.1	5 -0.08	-0.05	-0.23	-0.50
q7	12	I can easily start	5 -0.10	0.22	0.20	0.12	0.49
q63	68	A good night for me	5 0.3	0.04	-0.12	-0.17	-0.48
q56	61	I often and activel	5 0.03	0.26	0.12	0.40	0.46
q15	20	I tend to lead the	5 -0.30	0 -0.10	-0.01	0.05	0.44
q70	75	I bounce back quick	5 -0.38	3 -0.17	0.35	0.17	0.39
q71	76	I dont understand h	5 -0.2	-0.07	-0.17	0.23	0.39
q79	84	When given the choi	5 0.19	9 -0.13	0.23	-0.18	-0.38
q48	53	I tend to act on my	5 -0.03	3 -0.14	0.21	0.26	0.27

8.5 Comparing solutions

In order to compare the alternative solutions, we make use of the factor.congruence coefficient. We first compare the principal components solution to the factor solution, and then the factor solution to the cluster solution.

```
> round(factor.congruence(pc, fa4), 2)
```

PA1PA2PA3PA4PC11.000.18-0.25-0.29PC20.181.00-0.050.07PC3-0.24-0.041.00-0.02PC4-0.290.07-0.031.00

> round(factor.congruence(ic, fa4), 2)

PA1PA2PA3PA4C84-0.440.030.95-0.09C810.000.040.580.54C820.320.99-0.100.07C790.48-0.290.08-0.91C800.960.30-0.38-0.44

