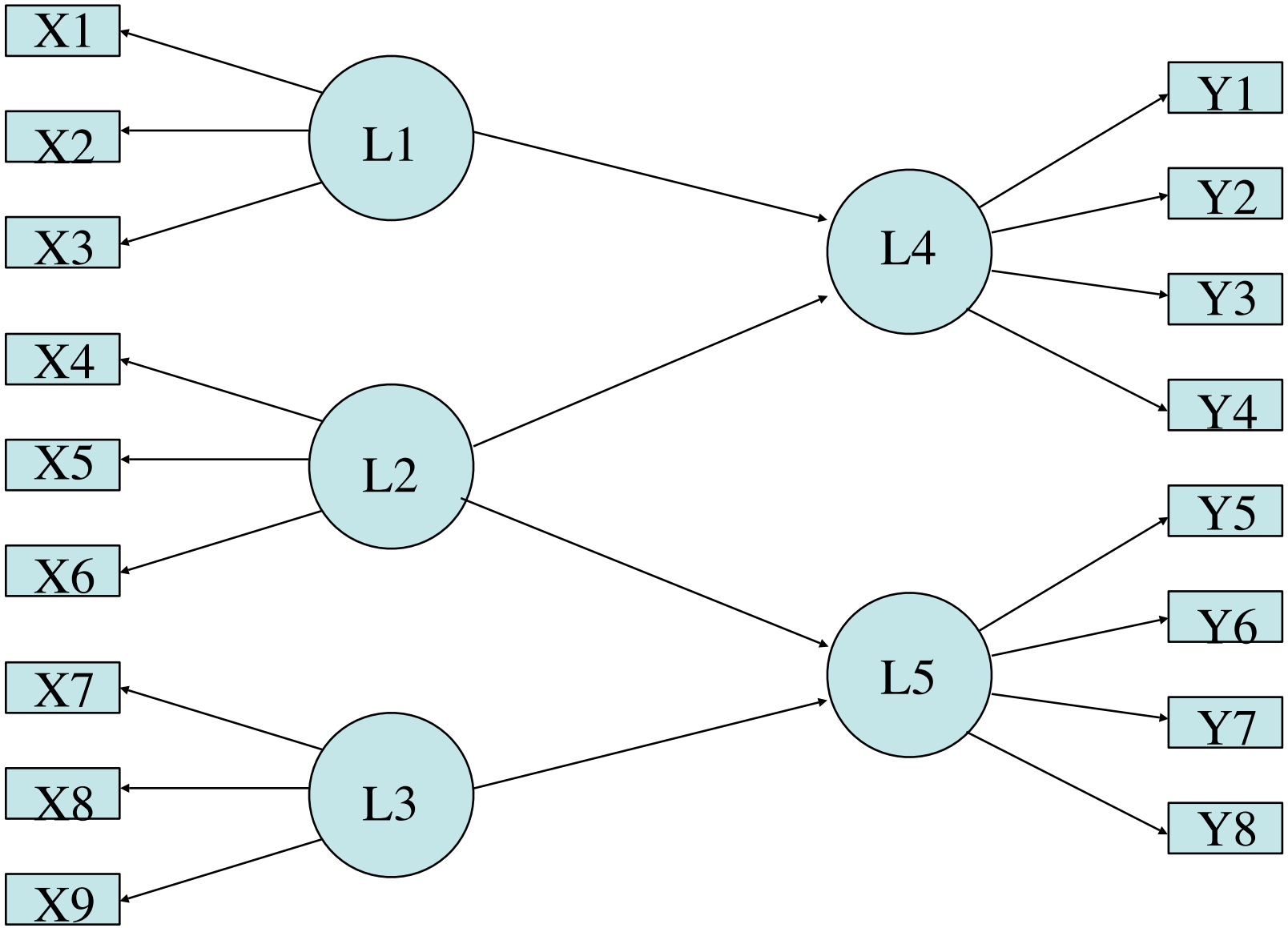
# Personality Research

- Current Theories of Personality
- Research Problems in Personality
- Measurement of personality
  - how do we measure the dimensions of personality
  - what do we measure when we measure
  - how well do we measure it

# Psychometric Theory: A conceptual Syllabus



# Psychometric Theory for Personality Research

## Basic Concepts of Variance, Covariance and Correlation

## with examples in R

# Basic statistics

- Central tendency
  - multiple measures, multiple ways of measuring
- Measures of dispersion
  - Single variables
  - composite variables
- Measures of relationship
  - Bivariate
  - Multivariate

# Use of R in personality research

- Powerful stats program available for all platforms
- Examples:
  - why.R.pdf in syllabus
  - http://personality-project.org/r/r.short.html
- How do you get it:
  - go to a CRAN download site (http://www.r-project.org)
  - get the psych package (from r-project.org)

# Estimates of Central Tendency

- Consider a set of observations $X = \{x_1, x_2 \ldots x_n\}$
- What is the best way to characterize this set
  - Mode: most frequent observation
  - Median: middle of ranked observations

**Mean:**

$$\textbf{Arithmetic} = \overline{X} = \sum_{1}^{n} (X_i)/N$$

$$\textbf{Geometric} = \sqrt[n]{\prod_{1}^{n} (X_i)}$$

$$\textbf{Harmonic} = \frac{N}{\sum_{1}^{n} (1/X_i)}$$

# Alternative expressions of mean

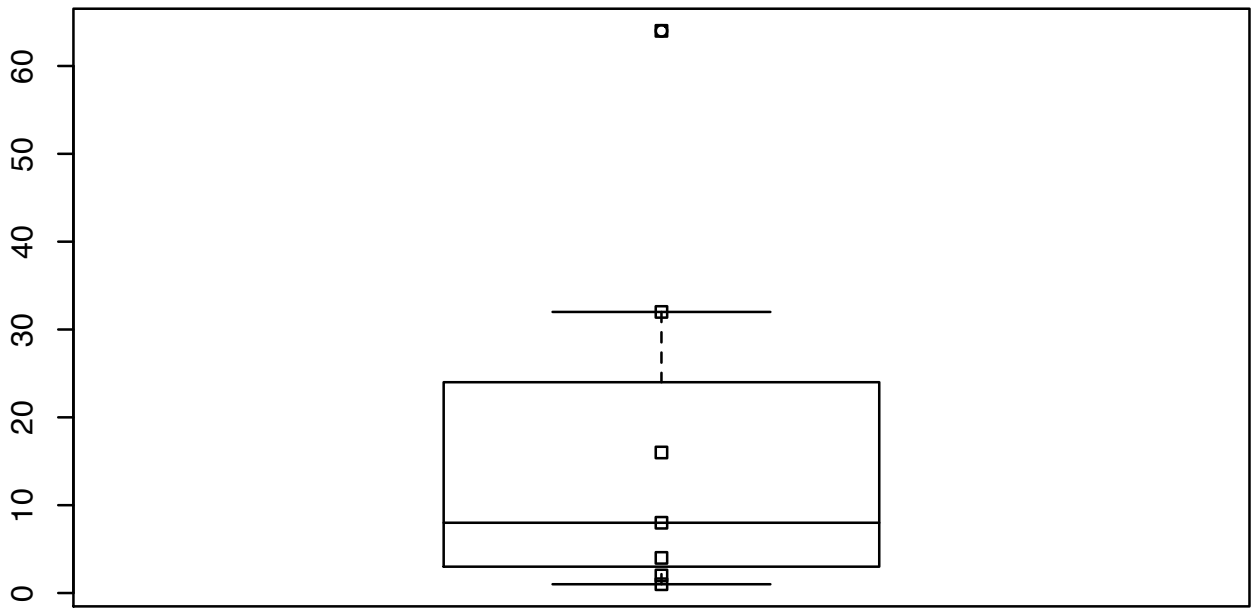
- Arithmetic mean = $\sum x_i/N$
- Alternatives are anti transformed means of transformed numbers
- Geometric mean = $\exp(\sum \ln(x_i)/N)$
  - (anti log of average log)
- Harmonic Mean = reciprocal of average reciprocal
  - $1/(\sum (1/x_i)/N)$

# Why all the fuss?

- Consider 1,2,4,8,16,32,64
- Median = 8
- Arithmetic mean = 18.1
- Trimmed mean (20% trim) = 12.4
- Geometric = 8
- Harmonic = 3.5
- trimmed harmonic (20% trim) = 5.16
- Which of these best captures the “average” value?

# Summary stats ( R code)

```
> x<-c(1,2,4,8,16,32,64)  #enter the data
> summary(x)  # simple summary
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   1.00    3.00    8.00   18.14   24.00   64.00
> boxplot(x)  #show five number summary
> stripchart(x,vertical=T,add=T)  #add in the points
```



# Consider two sets, which is bigger

| Subject | X | Y |
|---|---|---|
| 1 | 1 | 10 |
| 2 | 2 | 11 |
| 3 | 4 | 12 |
| 4 | 8 | 13 |
| 5 | 16 | 14 |
| 6 | 32 | 15 |
| 7 | 64 | 16 |
| Median | 8 | 13 |
| Arithmetic | 18.1 | 13 |
| Trimmed | 12.4 | 13 |
| geometric | 8.0 | 12.8 |
| harmonic | 3.5 | 12.7 |

# Summary stats (R code)

```
> x <- c(1,2,4,8,16,32,64)  #enter the data
> y <- seq(10,16)   #sequence of numbers from 10 to 16
> xy.df <- data.frame(x,y)  #create a "data frame"
> xy.df      #show the data
   x  y
1  1 10
2  2 11
3  4 12
4  8 13
5 16 14
6 32 15
7 64 16
> summary(xy.df)   #basic descriptive stats
       x                y
 Min.   : 1.00   Min.   :10.0
 1st Qu.: 3.00   1st Qu.:11.5
 Median : 8.00   Median :13.0
 Mean   :18.14   Mean   :13.0
 3rd Qu.:24.00   3rd Qu.:14.5
 Max.   :64.00   Max.   :16.0
```

# Summary stats (R code)

> describe(xy.df)

```
  var n  mean    sd median   mad min max range   se
x   1 7 18.14 22.94      8 10.38   1  64    63 8.67
y   2 7 13.00  2.16     13  2.97  10  16     6 0.82
```

round(geometric.mean(xy.df),2)

```
    x    y
 8.00 12.84
```

mean(xy.df,trim=.2)

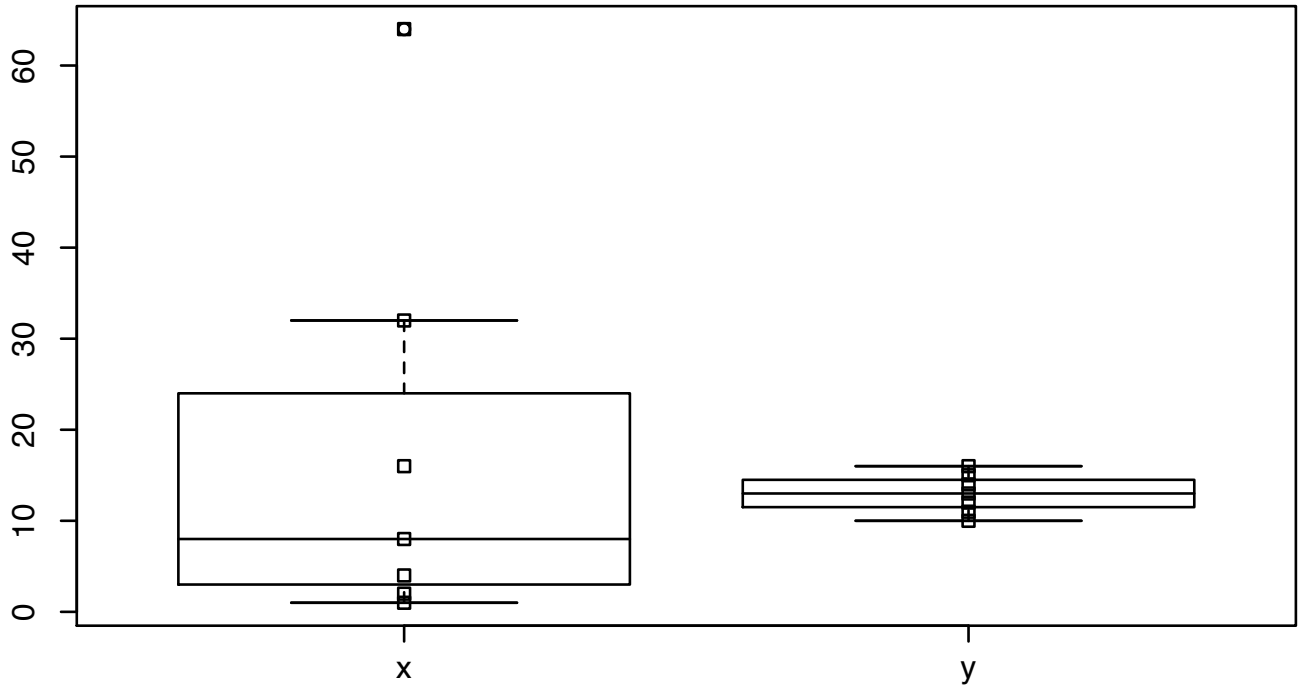
```
   x    y
12.4 13.0
```

round(harmonic.mean(xy.df),2)

```
    x    y
 3.53 12.69
```

# Box Plot (R)

boxplot(xy.df)  #show five number summary
stripchart(xy.df,vertical=T,add=T)  #add in the points



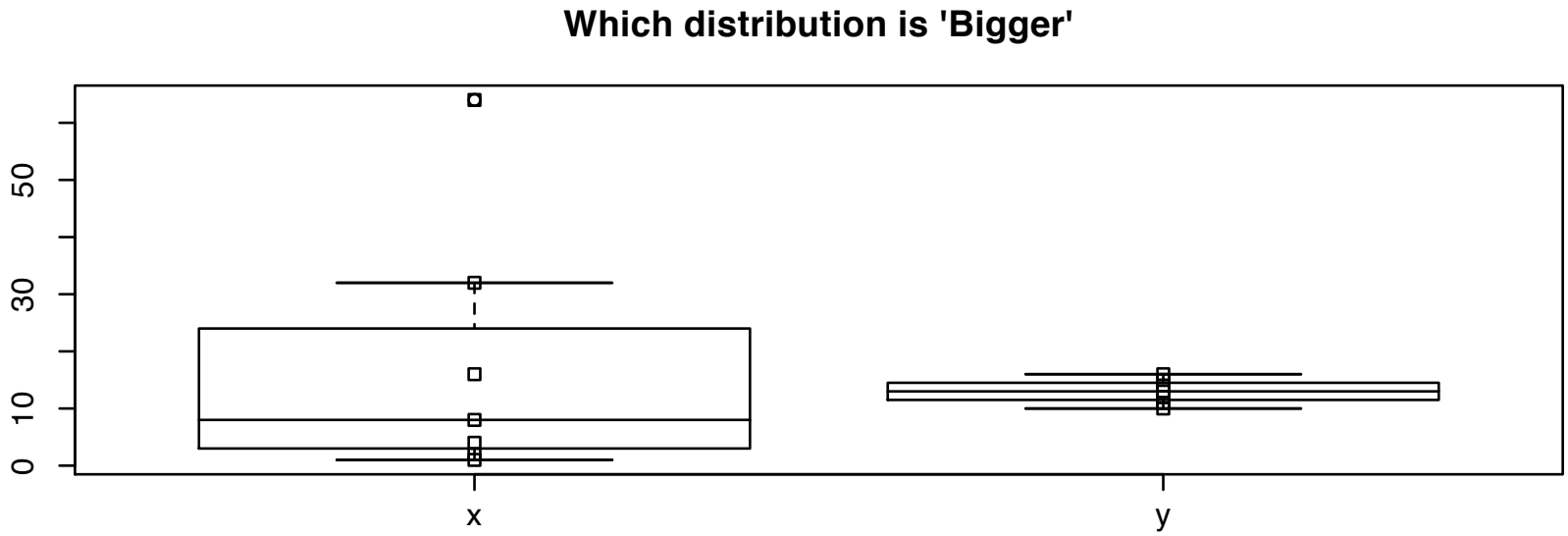
# The effect of log transforms
# Which group is "more"?

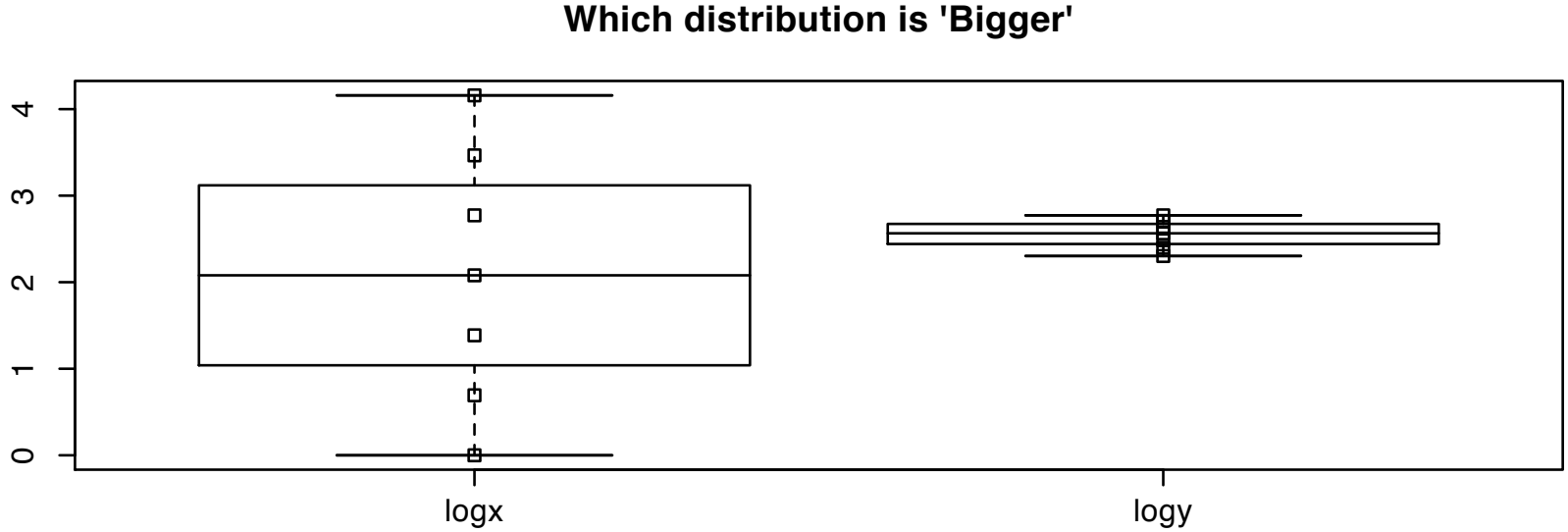
| X | Y | Log X | Log Y |
|---|---|---|---|
| 1 | 10 | 0.0 | 2.3 |
| 2 | 11 | 0.7 | 2.4 |
| 4 | 12 | 1.4 | 2.5 |
| 8 | 13 | 2.1 | 2.6 |
| 16 | 14 | 2.8 | 2.9 |
| 32 | 15 | 3.5 | 2.7 |
| 64 | 16 | 4.2 | 2.8 |

# Raw and log transformed which group is "bigger"?

| | X | Y | Log(X) | Log(Y) |
|---|---|---|---|---|
| Min | 1 | 10 | 0 | 2.30 |
| 1st Q. | 3 | 11.5 | 1.04 | 2.44 |
| Median | 8 | 13 | 2.08 | 2.57 |
| Mean | 18.1 | 13 | 2.08 | 2.26 |
| 3rd Q. | 24 | 14.5 | 3.12 | 2.67 |
| Max | 64 | 16 | 4.16 | 2.77 |

# The effect of a transform on means and medians



# Estimating central tendencies

- Although it seems easy to find a mean (or even a median) of a distribution, it is necessary to consider what is the distribution of interest.
- Consider the problems of the average length of psychotherapy, the average size of a class at NU, or the average velocity of cars on a highway.

# Estimating the mean time of therapy

- A therapist has 20 patients, 19 of whom have been in therapy for 26-104 weeks (median, 52 weeks), 1 of whom has just had their first appointment. Assuming this is her typical load, what is the average time patients are in therapy?
- Is this the average for this therapist the same as the average for the patients seeking therapy?

# Estimating the mean time of therapy

- 19 with average of 52 weeks, 1 for 1 week
  - Therapists average is (19*52+1*1)/20 = 49.5 weeks
  - Median is 52
- But therapist sees 19 for 52 weeks and 52 for one week so the average length is
  - ((19*52)+(52*1))/(19+52) = 14.6 weeks
  - Median is 1

# Estimating Class size

5 faculty members teach 20 courses with the following distribution: What is the average class size?

| Faculty member | 100 fr | 200 so-jr | 300 jr-sr | 400 grad | average |
|---|---|---|---|---|---|
| 1 | 10 | 20 | 10 | 10 | 12.5 |
| 2 | 10 | 20 | 10 | 10 | 12.5 |
| 3 | 10 | 20 | 10 | 10 | 12.5 |
| 4 | 20 | 100 | 20 | 10 | 37.5 |
| 5 | 400 | 100 | 100 | 100 | 175 |
| department | 90 | 52 | 30 | 28 | 50 |

# Estimating class size

- What is the average class size?
- If each student takes 4 courses, what is the average class size from the students' point of view?
- Department point of view: average is 50 students/class

| N | Size |
|---|---|
| 10 | 10 |
| 5 | 20 |
| 4 | 100 |
| 1 | 400 |

# Estimating Class size

| Faculty member | A | B | C | D | average |
|---|---|---|---|---|---|
| 1 | 10 | 20 | 10 | 10 | 12.5 |
| 2 | 10 | 20 | 10 | 10 | 12.5 |
| 3 | 10 | 20 | 10 | 10 | 12.5 |
| 4 | 20 | 100 | 20 | 10 | 37.5 |
| 5 | 400 | 100 | 100 | 100 | 175 |
| department | 90 | 52 | 30 | 28 | 50 |

# Estimating Class size (student weighted)

| Faculty member | A | B | C | D | average |
|---|---|---|---|---|---|
| 1 | 10 | 20 | 10 | 10 | 14 |
| 2 | 10 | 20 | 10 | 10 | 14 |
| 3 | 10 | 20 | 10 | 10 | 14 |
| 4 | 20 | 100 | 20 | 10 | 73 |
| 5 | 400 | 100 | 100 | 100 | 271 |
| Student | 357 | 82 | 71 | 74 | 206 |

# Estimating class size

Department perspective:

20 courses, 1000 students => average = 50

Student perspective: 1000 students enroll in classes with an average size of 206!

Faculty perspective: chair tells prospective faculty members that median faculty course size is 12.5, tells the dean that the average is 50 and tells parents that most upper division courses are small.

# Traffic Flow

- Three lanes of traffic, uniformly distributed
  - one lane is traveling at 10 mph
  - one lane is travelling at 20 mph
  - one lane is traveling at 30 mph
- What is the average velocity of cars?
- What is the median velocity?

# Traffic Flow:
# But officer, I wasn't speeding

- Three lanes of traffic, uniformly distributed
  - one lane is traveling at 10 mph
  - one lane is travelling at 20 mph
  - one lane is traveling at 30 mph
- Assume cars are spaced a mile apart
  - Average = 30*30 +20*20 +10 *10 = 1400/60 =
    - 23.3
  - Median is 50th percentile -- mid point between 20 and 30 = 25

# Average Velocity

- On a 100 mile trip from Chicago to Milwaukee, you drive the first 50 miles at 30 miles/hour and the second half at 60 miles/hour. What is your average velocity?
- A race car driver has to average 90 miles an hour for two laps of a one mile track. He does the first lap at 45 mph. How fast must he drive the second lap?

# Velocity leads to time weighting

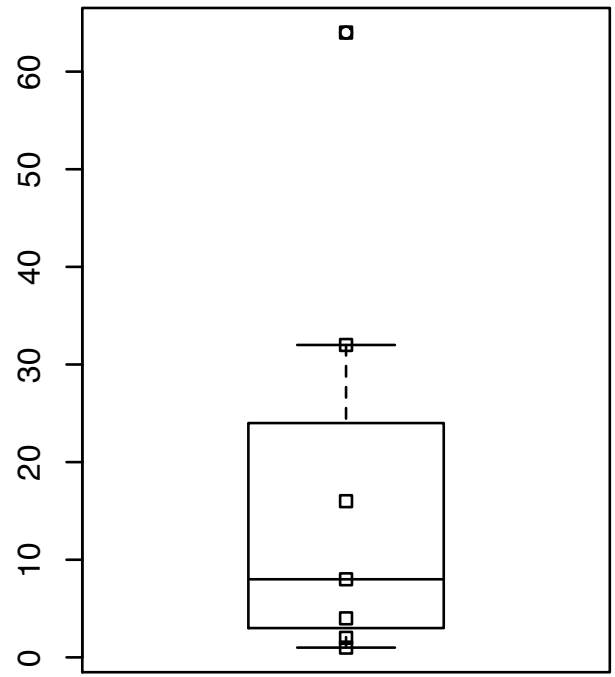
- A trip to Milwaukee:
  - 50 miles at 30 mph = 1.66 hours
  - 50 miles at 60 mph = .833 hours.
  - Average is (1.66* 30 + .833 * 60)/2.5 = 40 mph
- Race car driver
  - First lap at 45 => 1.33 minutes
  - Total time allowed = 120 secs/90 =1.33 minutes
  - driver can not average 90!

# Measures of dispersion

- Range (maximum - minimum)
- Interquartile range (75% - 25%)
- Deviation score $x_i = X_i$-Mean
- Median absolute deviation from median
- Variance = $\sum x_i^2/(N-1)$ = mean square
- Standard deviation sqrt (variance ) =sqrt($\sum x_i^2/(N-1)$)

# Robust measures of dispersion

- The 5-7 numbers of a box plot
- Max
- Top Whisker
- Top quartile (hinge)
- Median
- Bottom Quartile (hinge)
- Bottom Whisker
- Minimum



# Raw scores, deviation scores and Standard Scores

- Raw score for $i_{th}$ individual $X_i$
- Deviation score $x_i = X_i$-Mean X
- Standard score = $x_i/s_x$
- Variance of standard scores = 1
- Mean of standard scores = 0
- Standard scores are unit free index

# Transformations of scores

- Mean of (X+C) = Mean(X) + C
- Variance (X+C) = Variance(X)
- Variance (X*C) = Variance(X) *$C^2$
- Coefficient of variation = sd/mean

# Typical transformations

| | Mean | Standard Deviation |
|---|---|---|
| Raw data | $X. = \sum X/n$ | $Sqrt(\sum (X-X.)^2)/(n-1) =$ $s_x = Sqrt(\sum x^2)/(n-1)$ |
| deviation score | 0 | $s_x$ |
| Standard score | 0 | 1 |
| “IQ” | 100 | 15 |
| “SAT” | 500 | 100 |
| “T-Score” | 50 | 10 |
| “stanine” | 5 | 1.5 |

# Variance of Composite

| | X | Y |
|---|---|---|
| X | Variance $_X$ | Covariance $_{XY}$ |
| Y | Covariance $_{XY}$ | Variance $_Y$ |

Variance $_{(X+Y)}$ = Var $_X$ + Var $_Y$ + 2 Cov $_{XY}$

# Variance of Composite

| | X | Y |
|---|---|---|
| X | $\sum x_i^2/(N-1)$ | $\sum x_i y_i /(N-1)$ |
| Y | $\sum x_i y_i /(N-1)$ | $\sum yi\ 2/(N-1)$ |

$$\text{Var}_{(X+Y)} = \sum (x_i+y_i)^2/(N-1) = \sum x_i^2/(N-1) + \sum y_i^2/(N-1) + 2\sum x_i y_i /(N-1)$$

# Consider the following problem

- If you have a GRE V of 700 and a GRE Q of 700, how many standard deviations are you above the mean GRE (V+Q)?
- Need to know the Mean and Variance of V, Q, and V+Q

| | GRE V | GRE Q | GRE V+Q |
|---|---|---|---|
| Mean | 500 | 500 | 1000 |
| SD | 100 | 100 | ? |

# Variance of GRE (V+Q)

| | GREV | GRE Q |
|---|---|---|
| GREV | 10,000 | 6,000 |
| GRE Q | 6,000 | 10,000 |

Variance of composite = 32,000 => s.d. composite = 179

# Variance of GRE (V+Q)

| | $GRE_V$ | $GRE_Q$ | $GRE_{V+Q}$ |
|---|---|---|---|
| Mean | 500 | 500 | 1000 |
| SD | 100 | 100 | 179 |

# Standard score on composite

| | GRE $_V$ | GRE $_Q$ | GRE $_{V+Q}$ |
|---|---|---|---|
| mean | 500 | 500 | 1000 |
| sd | 100 | 100 | 179 |
| raw score | 700 | 700 | 1400 |
| z score | 2 | 2 | 2.23 |
| percentile | 97.7 | 97.7 | 98.7 |

# Variance of composite of n variables: generalization of variance of x+y

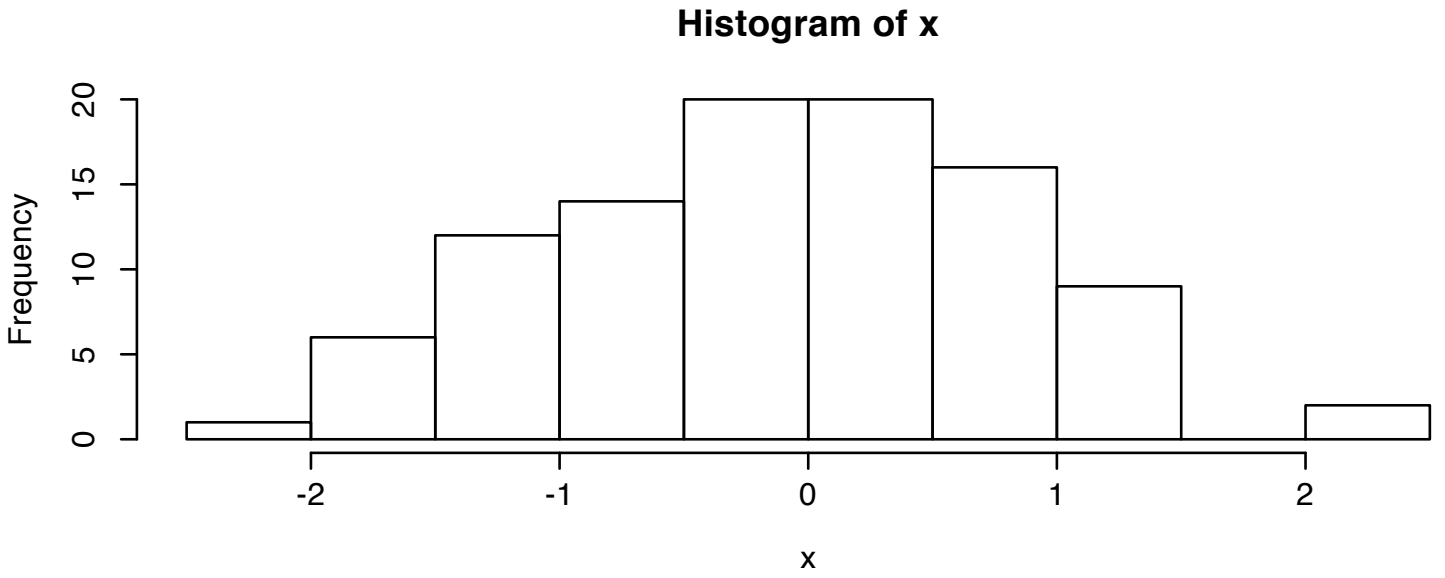
| | $X_1$ | $X_2$ | … | $X_i$ | $X_j$ | … | $X_n$ |
|---|---|---|---|---|---|---|---|
| $X_1$ | $Vx_1$ | | | | | | |
| $x_2$ | $Cx_1x_2$ | $Vx2$ | | | | | |
| … | | | … | | | | |
| $X_i$ | $Cx_1x_i$ | $Cx_2x_i$ | | $Vx_i$ | | | |
| $X_j$ | $Cx_1x_j$ | $Cx_2x_j$ | | $Cx_ix_i$ | $Vx_j$ | | |
| … | | | | | | … | |
| $X_n$ | $Cx_1x_n$ | $Cx_2x_n$ | | $Cx_ix$ | $Cx_jx$ | | $Vxn$ |

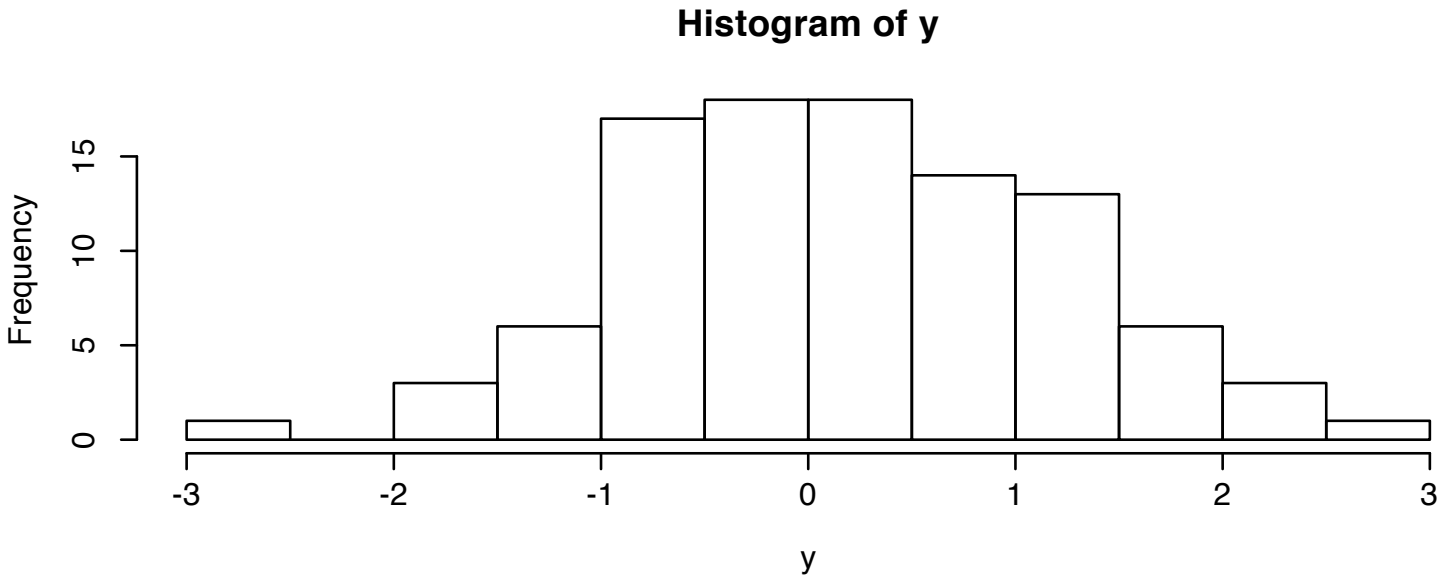
Variance of composite of n items has n variances and n*(n-1) covariances

# Variance, Covariance, and Correlation

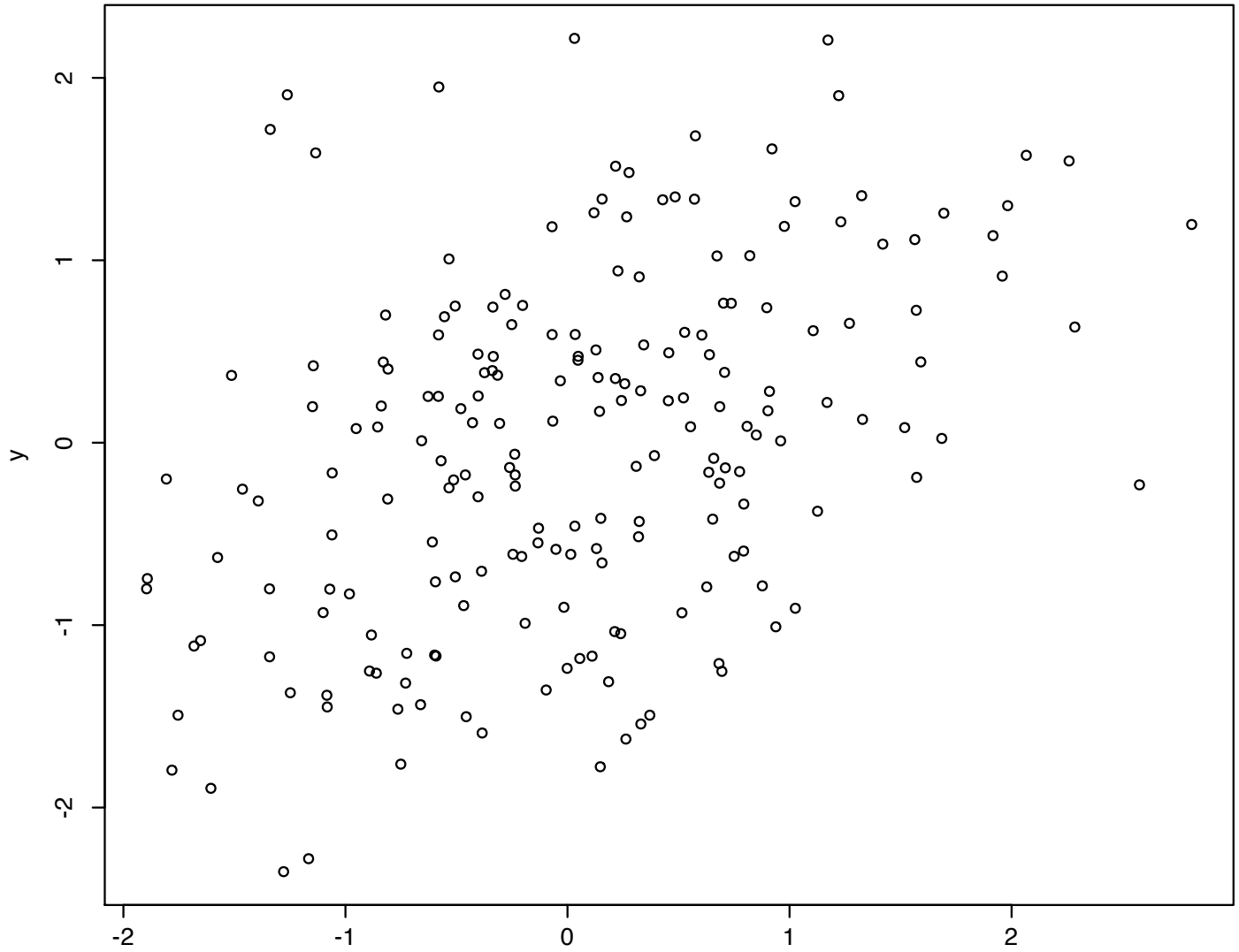
- Given two variables, X and Y, can we summarize how they interrelate?
- Given a score $x_i$, what does this tell us about $y_i$
- What is the amount of uncertainty in Y that is reduced if we know something about X.
- Example:  the effect of daily temperature upon amount of energy consumed per day
- Example: the relationship between anxiety and depression

# Distributions of two variables

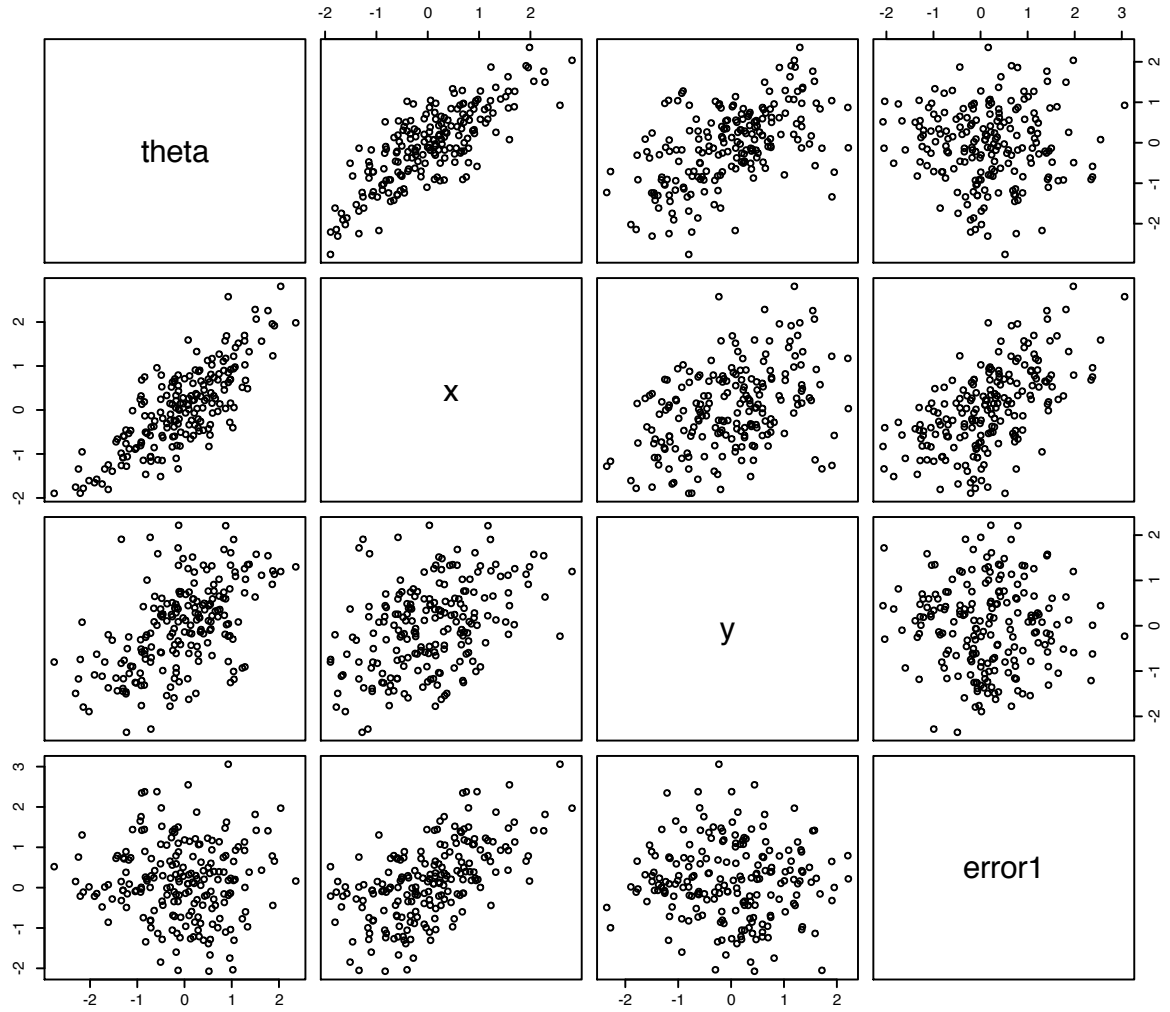


# Joint distribution of X and Y



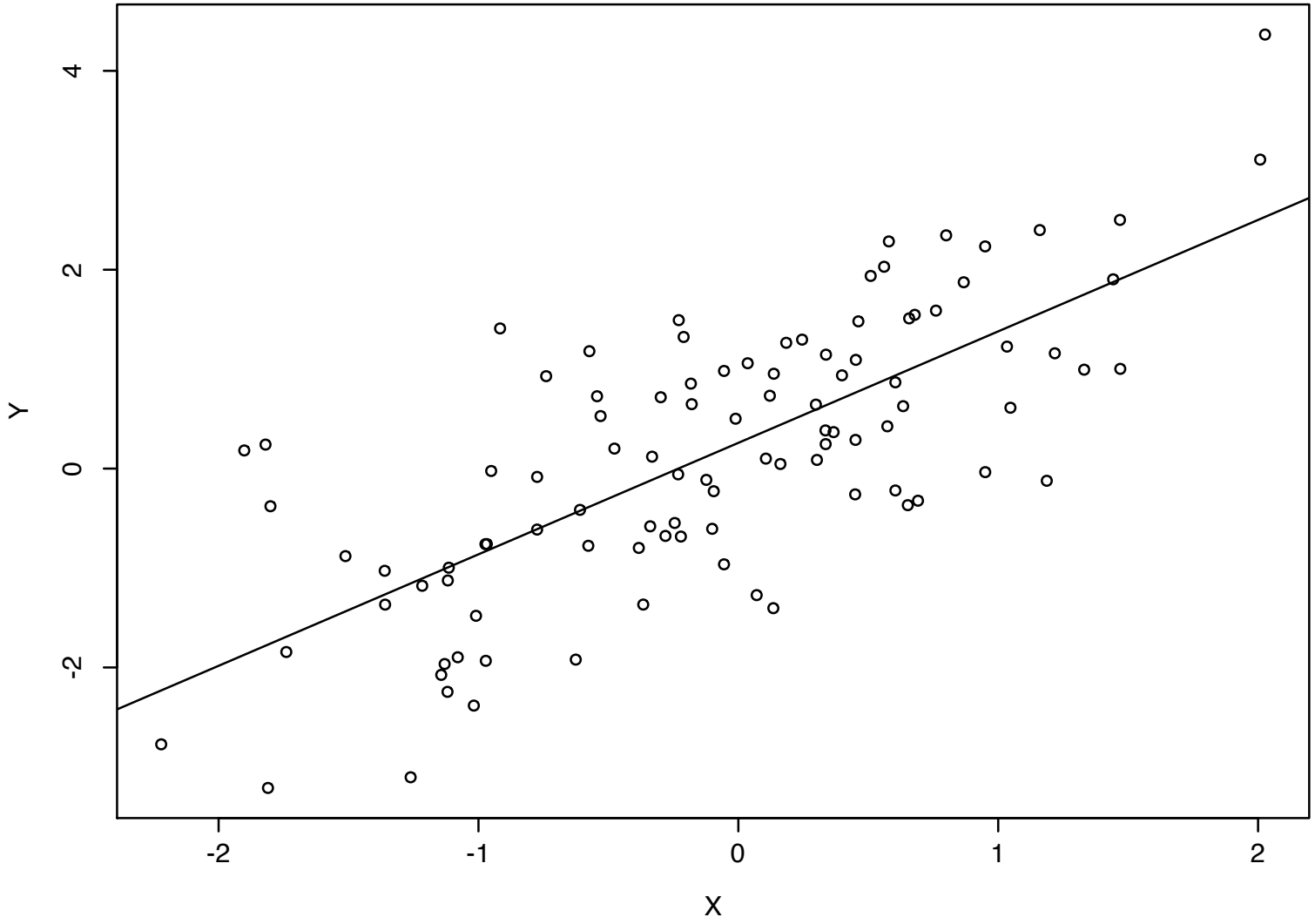
# The problem of summarizing several bivariate relationships



# Predicting Y from X

- First order approximation: predict mean Y for all y
- Second order approximation: predict $y_i$ deviates from mean Y as linear function of deviations of $x_i$ from mean X
- $Y_i = Y. + b_{xy}(X_i - X.)$ or $y_i = b_{xy}(x_i)$
- What is the best value of $b_{xy}$?

# Predicting Y from X



# The problem of predicting y from x:

- Linear prediction $y=bx+c$ $Y=b(X-M_x)+M_y$
- error in prediction = predicted y - observed y
- problem is to minimize the squared error of prediction
- minimize the error variance = $V_e = [\sum(y_p-y_o)^2]/(N-1)$
- $V_e=V_{(bx-y)} = \sum (bx-y)^2 /(N-1)=$
- $\sum(b^2x^2-2bxy+y^2) /(N-1)=$
- $b^2\sum x^2/(N-1)-2b\sum xy/(N-1)+\sum y^2/(N-1)\Rightarrow$
- $V_e = b^2V_x-2bC_{xy}+V_y$

- $V_e$ is minimized when the first derivative (w.r.t. b) = 0 ==>
- when $2bV_x-2C_{xy}=0$ ==>
- $b_{y.x}=C_{xy}/V_x$

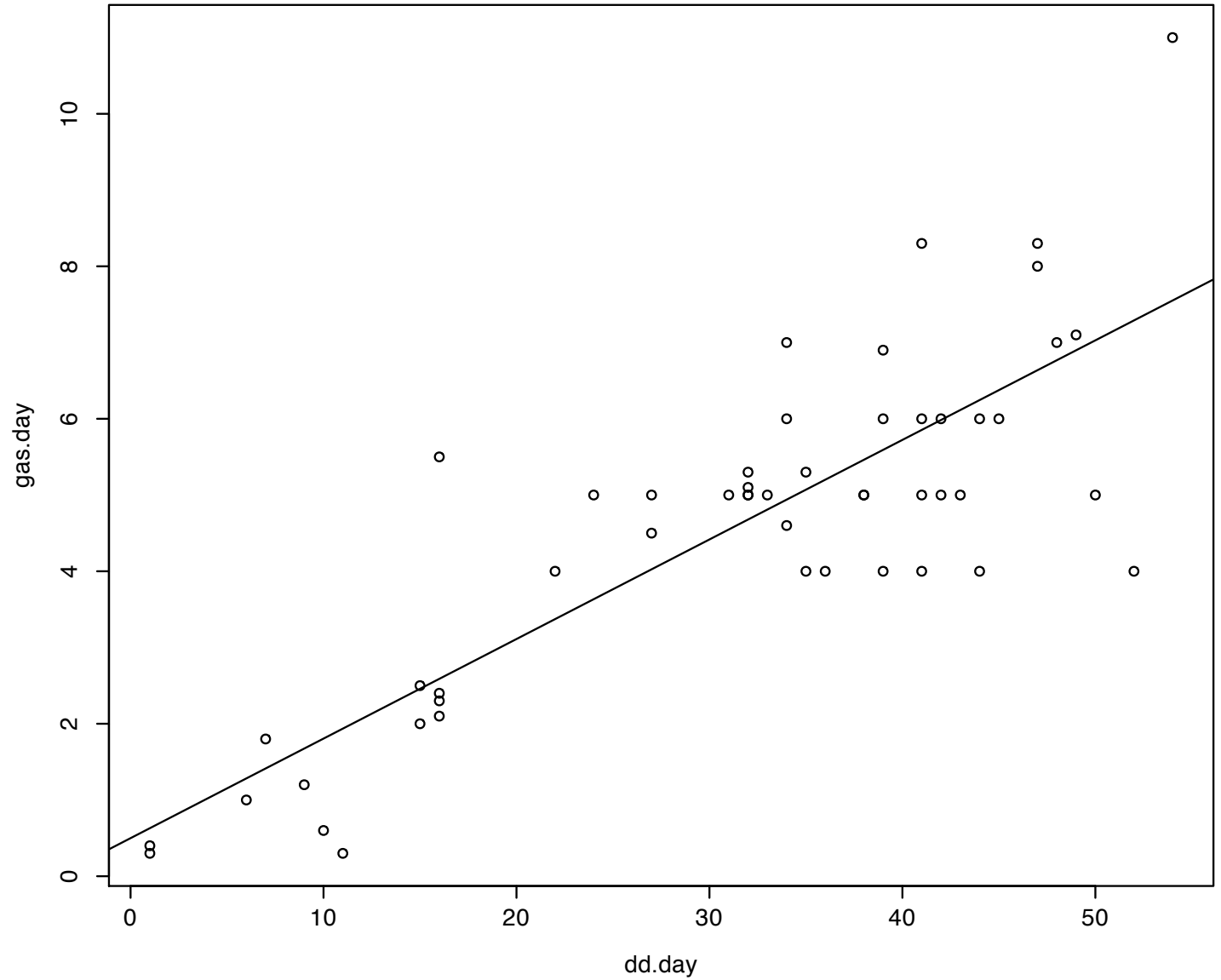
# Measures of relationship

- Regression $y = bx + c$
  - $b_{y.x} = Cov_{xy} / Var_{x}$ $\quad\quad$ $b_{x.y} = Cov_{xy} / Var_{y}$
- Correlation
  - $r_{xy} = Cov_{xy} / sqrt(V_x * V_y)$
  - Pearson Product moment correlation
    - Spearman (ppmc on ranks)
    - Point biserial (x is dichotomous, y continuous)
    - Phi (x, y both dichotomous)

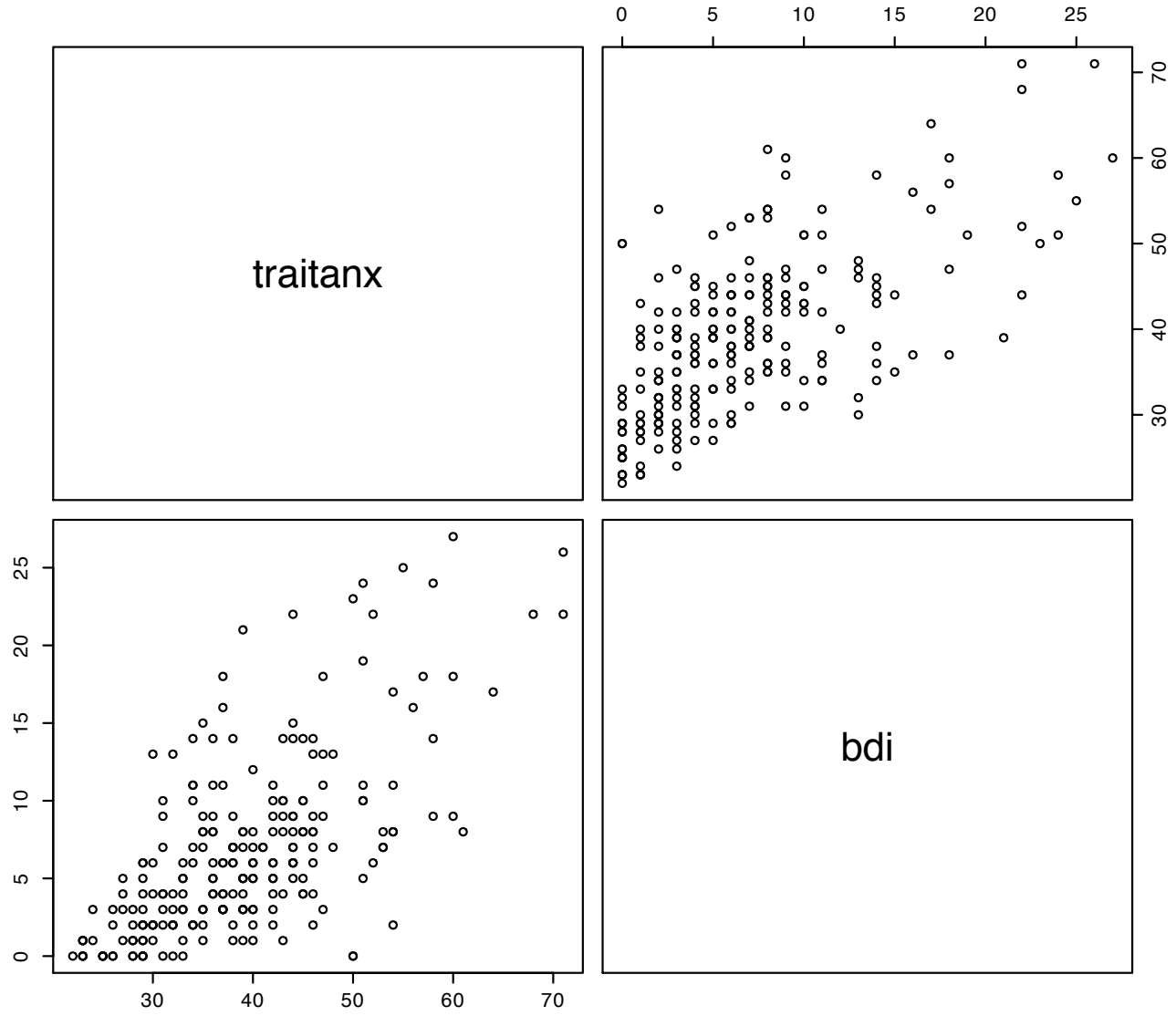
# Correlation and Regression

- Regression slope is in units of DV and IV
  - regression implies IV -> DV
    - (gas consumption as function of outside temp)
- Correlation is unit free index of relationship
  - (geometric) average of two regression slopes
  - slope of standardized IV regression on standardized DV => unit free index
  - a measure of goodness of fit of regression

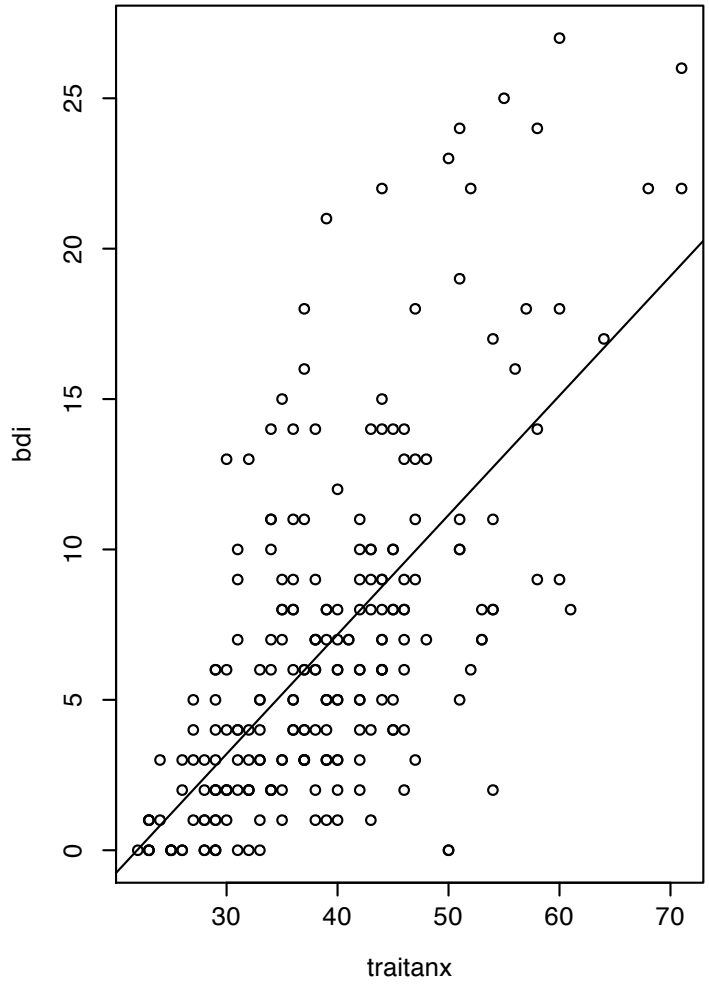
# Gas Consumption by degree day (daily data)

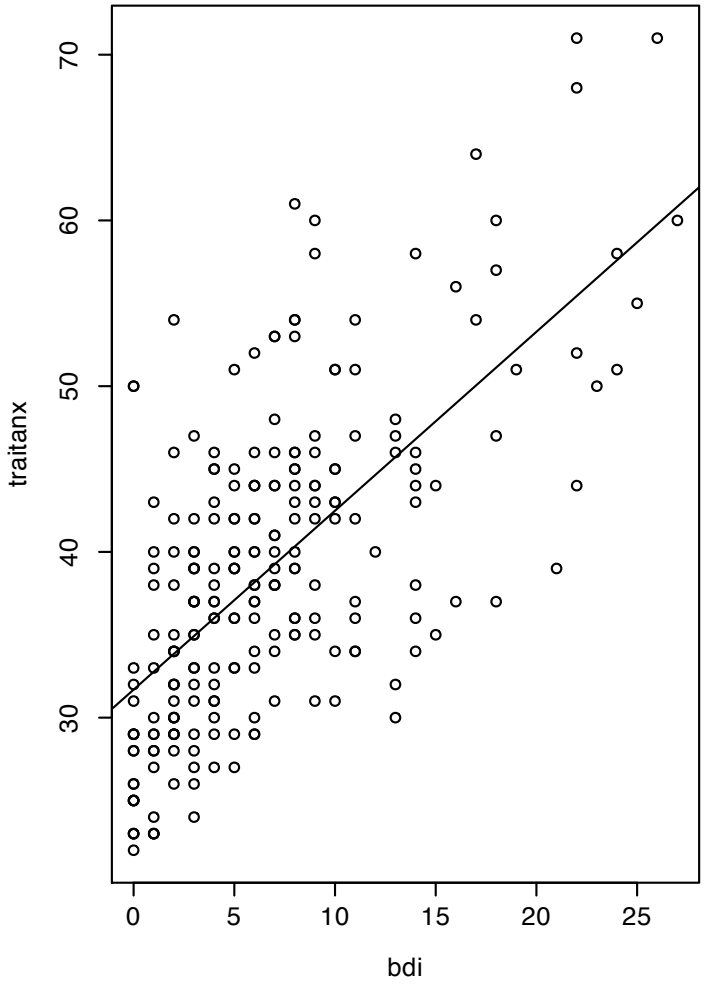


# Beck Depresion x Trait Anxiety (raw)

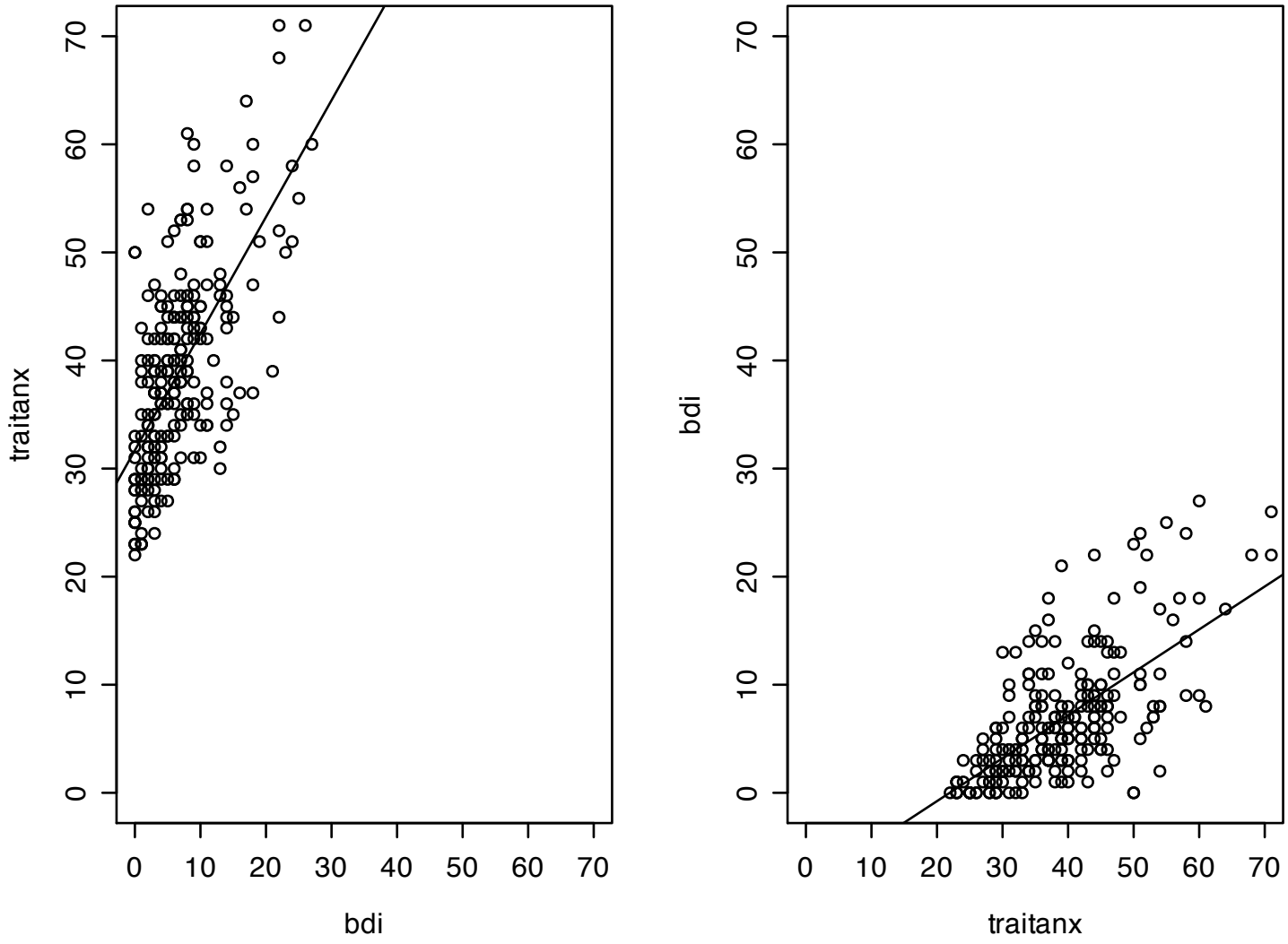


# BDI x Trait Anx (raw)

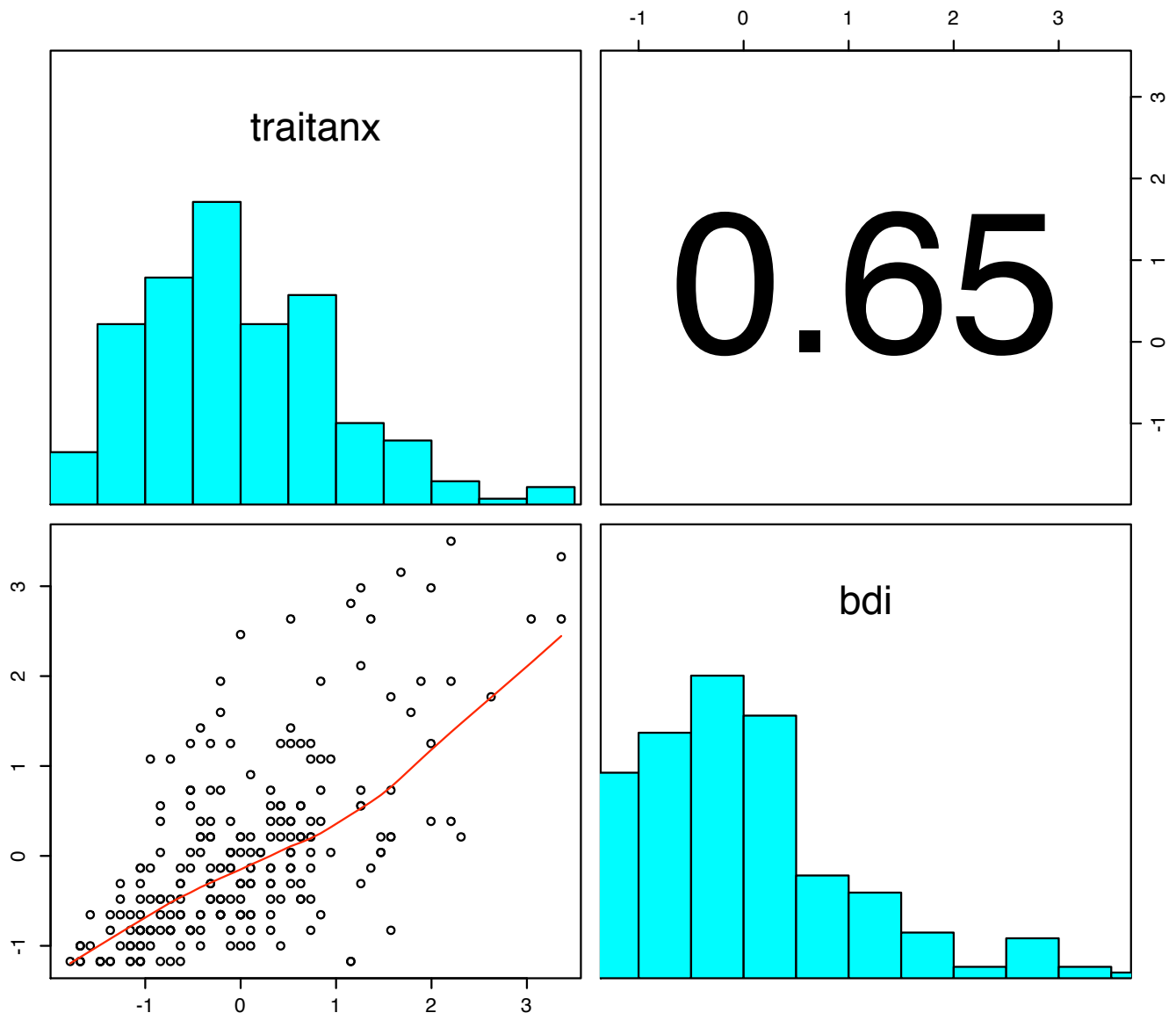


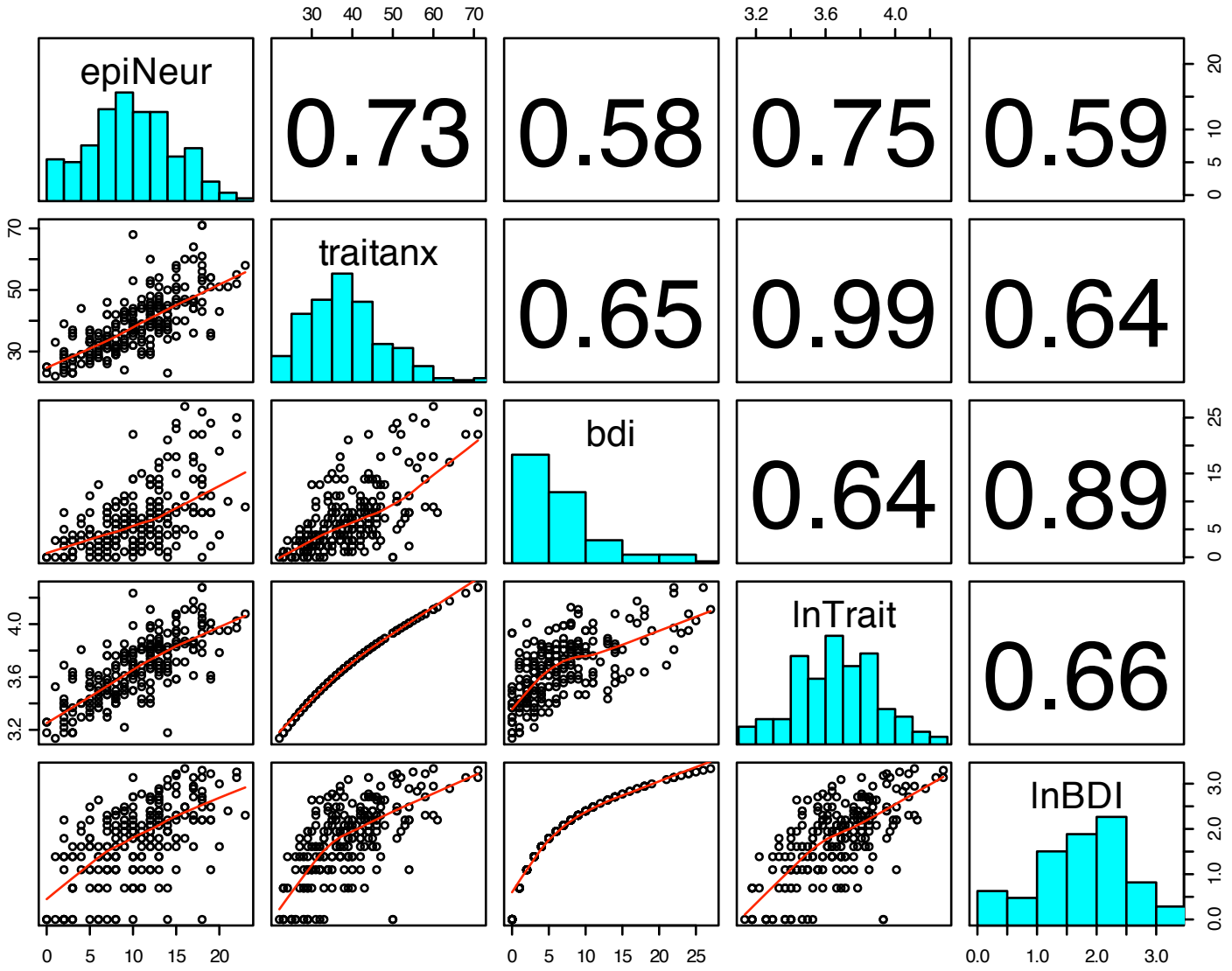
# Regression lines depend upon scale



# Beck Depression * Trait Anxiety z score



# Transforming can help



# Alternative forms of r

$r = cov_{xy}/Sqrt(V_x * V_y) =$

$(\sum xy/N)(sqrt(\sum x^2/N * \sum y^2/N) = (\sum xy)(sqrt(\sum x^2 * \sum y^2)$

| Correlation | X | Y |
|---|---|---|
| Pearson | Continuous | Continuous |
| Spearman | Ranks | ranks |
| Point biserial | Dichotomous | Continuous |
| Phi | Dichotomous | Dichotomous |
| *Biserial* | Dichotomous (assumed normal) | Continuous |
| *Tetrachoric* | Dichotomous (assumed normal) | Dichotomous (assumed normal |
| *Polychoric* | categorical (assumed normal) | categorical (assumed normal) |

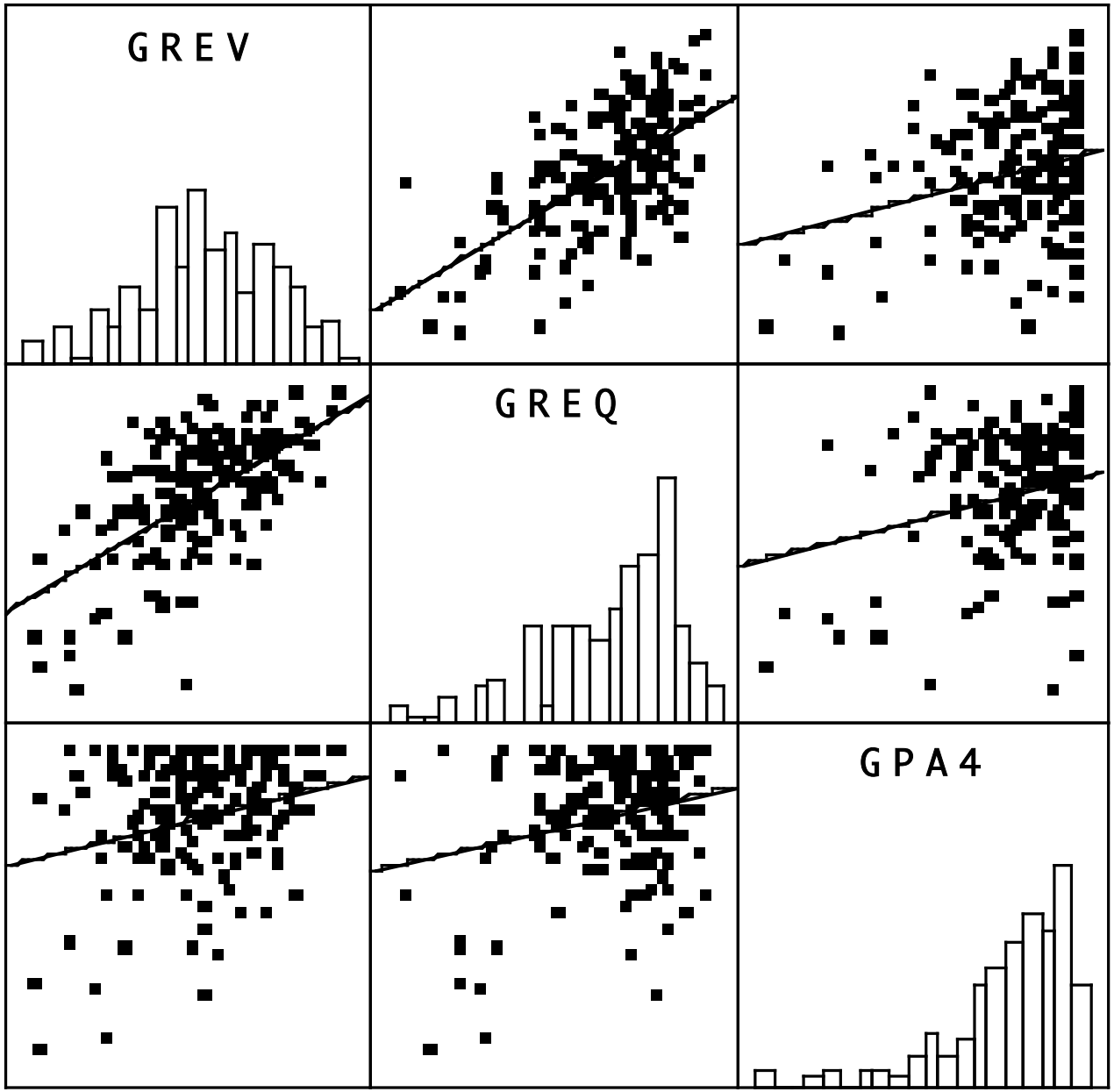
# Correlation Matrix: GRE V, Q, GPA

PEARSON CORRELATION MATRIX

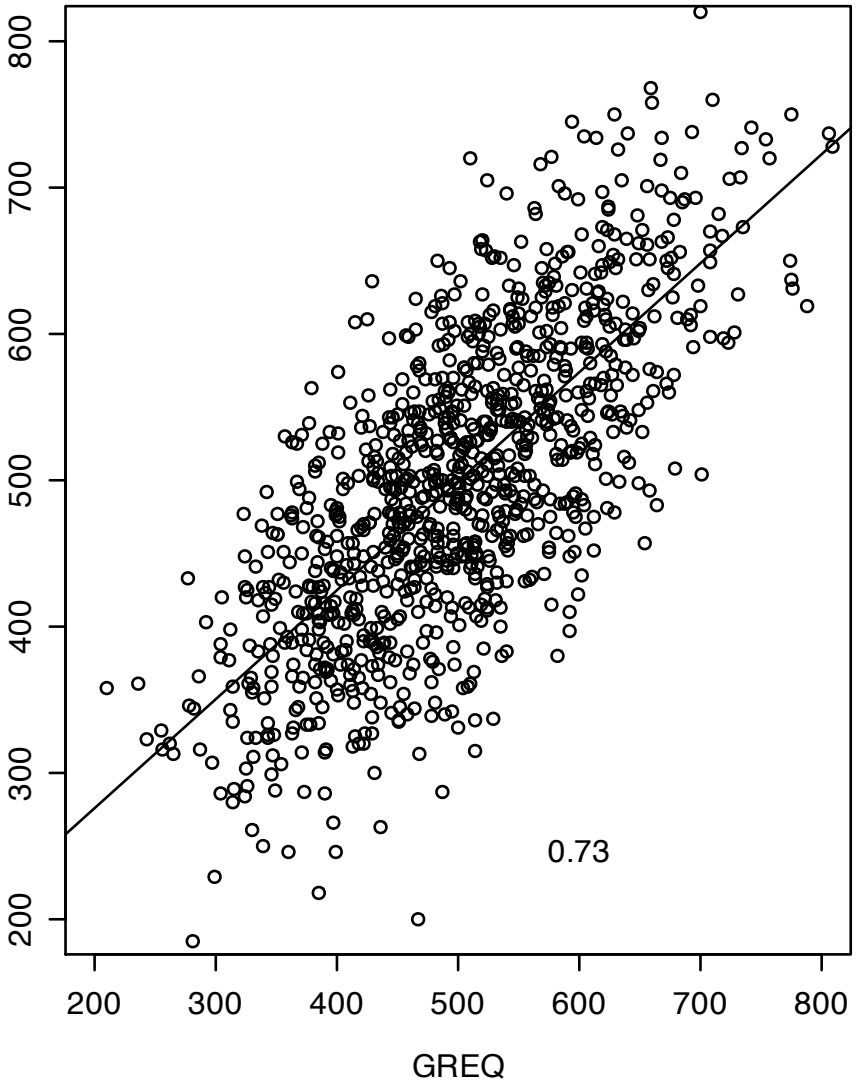
| | GREV | GREQ | GPA4 |
|---|---|---|---|
| GREV | 1.00 | | |
| GREQ | 0.61 | 1.00 | |
| GPA4 | 0.27 | 0.25 | 1.00 |

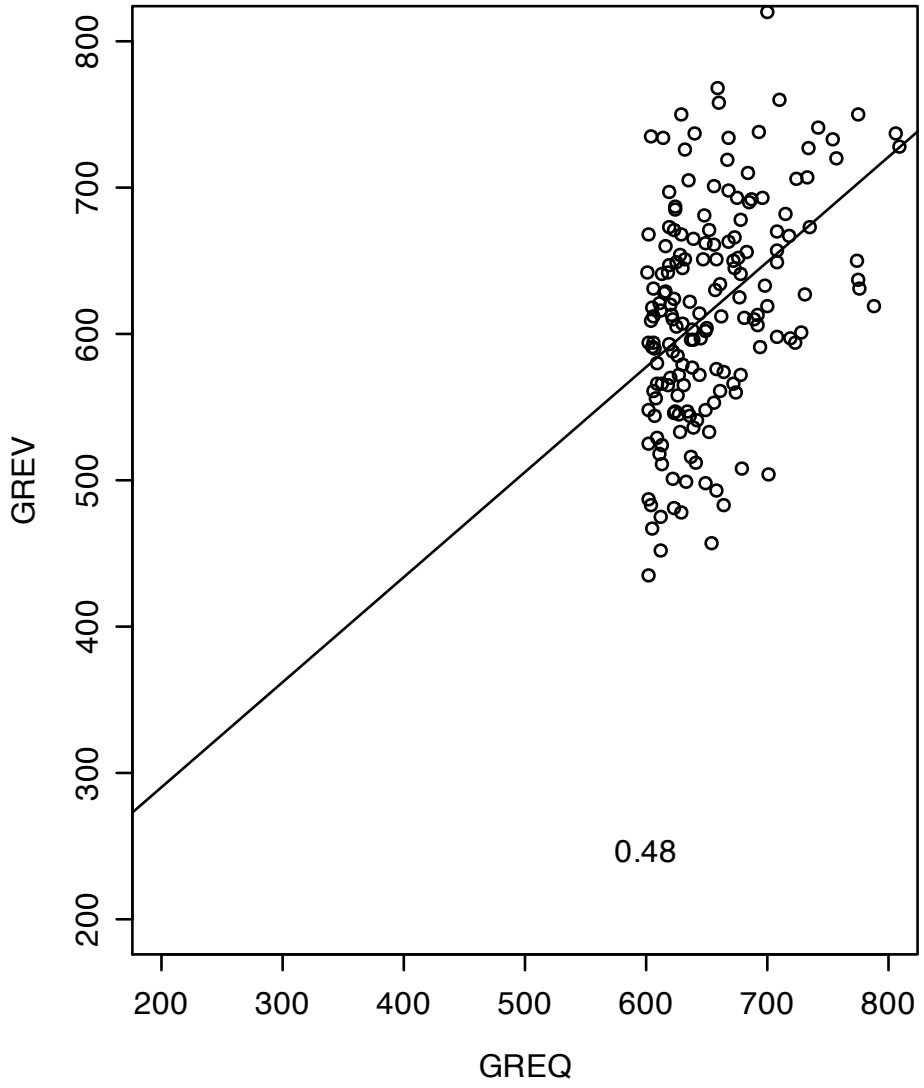
NUMBER OF OBSERVATIONS: 163

# SPLOM of GRE V, Q, GPA



# The effect of restriction of range on regression slopes vs. correlations



# Caution with correlation

Consider 8 variables with means:

| x1 | x2 | x3 | x4 | y1 | y2 | y3 | y4 |
|---|---|---|---|---|---|---|---|
| 9.0 | 9.0 | 9.0 | 9.0 | 7.5 | 7.5 | 7.5 | 7.5 |

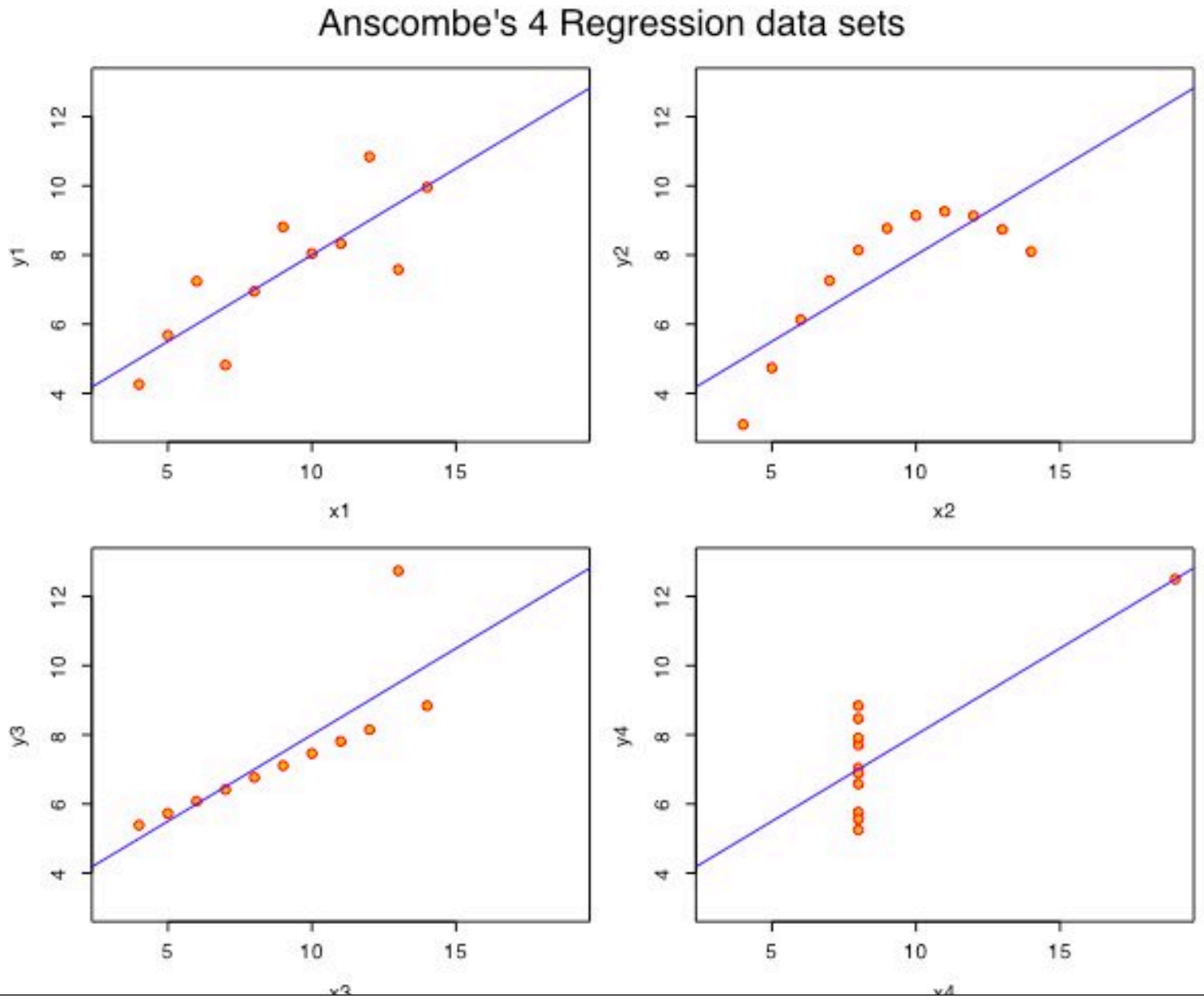
and Standard deviations

| x1 | x2 | x3 | x4 | y1 | y2 | y3 | y4 |
|---|---|---|---|---|---|---|---|
| 3.32 | 3.32 | 3.32 | 3.32 | 2.03 | 2.03 | 2.03 | 2.03 |

and correlations between xi and yi of

| | | | |
|---|---|---|---|
| 0.82 | 0.82 | 0.82 | 0.82 |

# Caution with Correlation



# Correlation: Alternative meanings

1) Slope of regression ($b_{xy} = C_{xy}/V_x$) reflects units of x and y but the correlation {$r = C_{xy}/(S_xS_y)$} is unit free.

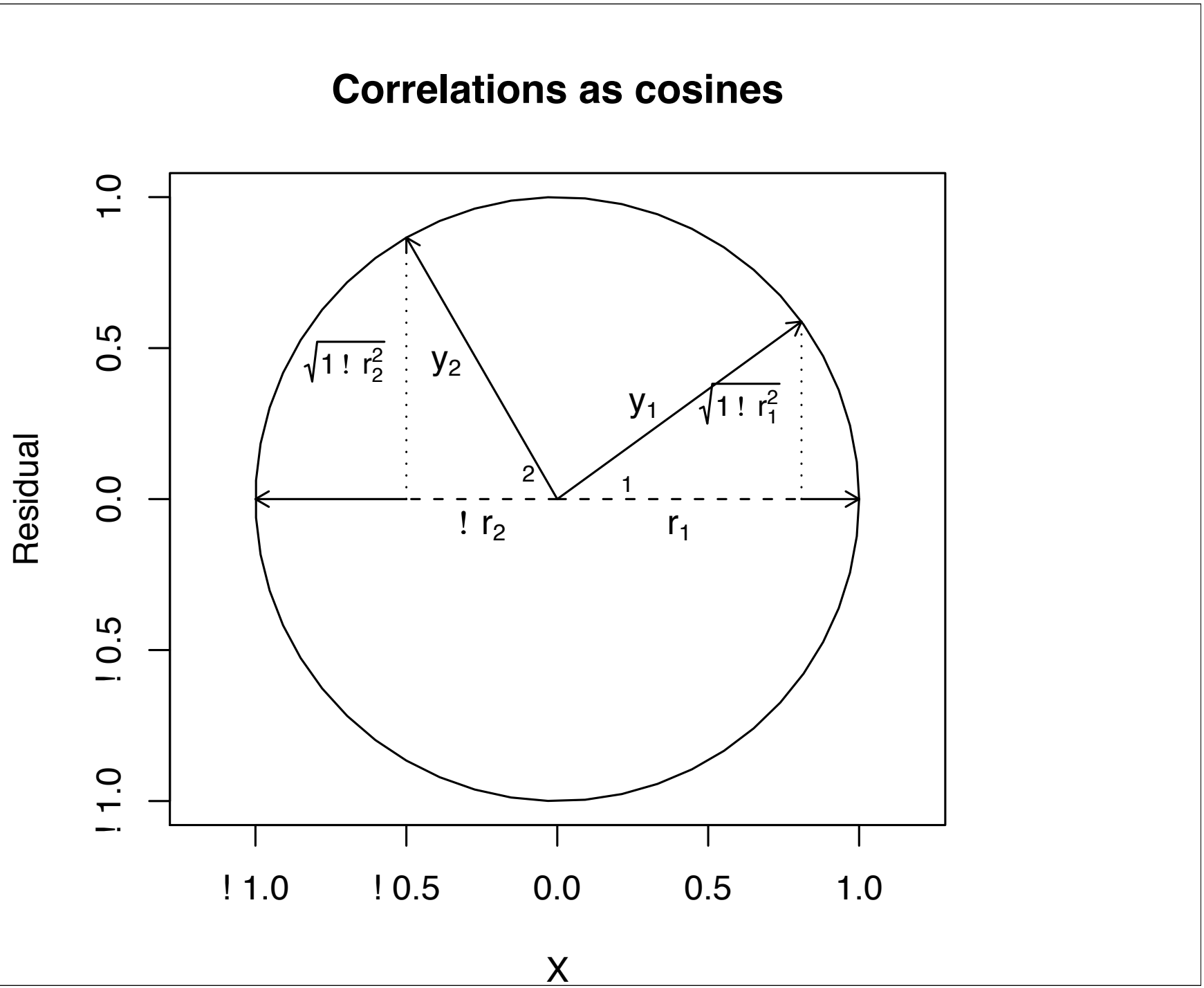
2) Geometrically, r = cosine (angle between test vectors)

3) Correlation as prediction:
Let $y_p$ = predicted deviation score of y = predicted Y - M

$y_p = b_{xy}x$ and $b_{xy} = C_{xy}/V_x = rS_y/S_x$ ==> $y_p/S_y = r(x/S_x)$==>

predicted z score of y ($z_{yp}$ ) = $r_{xy}$ * observed z score of x ($z_x$)

predicted z score of x ($z_{xp}$ ) = $r_{xy}$ * observed z score of y ($z_y$)


Correlations as cosines
Residual
X
1.0
0.5
0.0
! 0.5
! 1.0
! 1.0
! 0.5
0.0
0.5
1.0
$\sqrt{1 ! r_2^2}$
$y_2$
$y_1$
$\sqrt{1 ! r_1^2}$
2
1
! $r_2$
$r_1$

# Correlation as goodness of fit

Amount of error variance (residual or unexplained variance) in y given x and r

$V_e = \sum e^2/N = \sum y\text{-}bx)^2/N = \sum\{y\text{-}(r*S_y*x/S_x)\}^2 =$
$V_y + V_y * r^2 - 2(r * S_y * C_{xy})/S_x$
(but $S_y * C_{xy}/S_x = V_y * r$ )
$V_y + V_y * r^2 - 2(r^2 * V_y) = V_y(1\text{-}r^2)$ ==>
$V_e = V_y(1\text{-}r^2)$ <==> $V_{yp} = V_y(r^2)$

Residual Variance = Original Variance * $(1\text{-}r^2)$

Variance of predicted scores = original variance * $r^2$

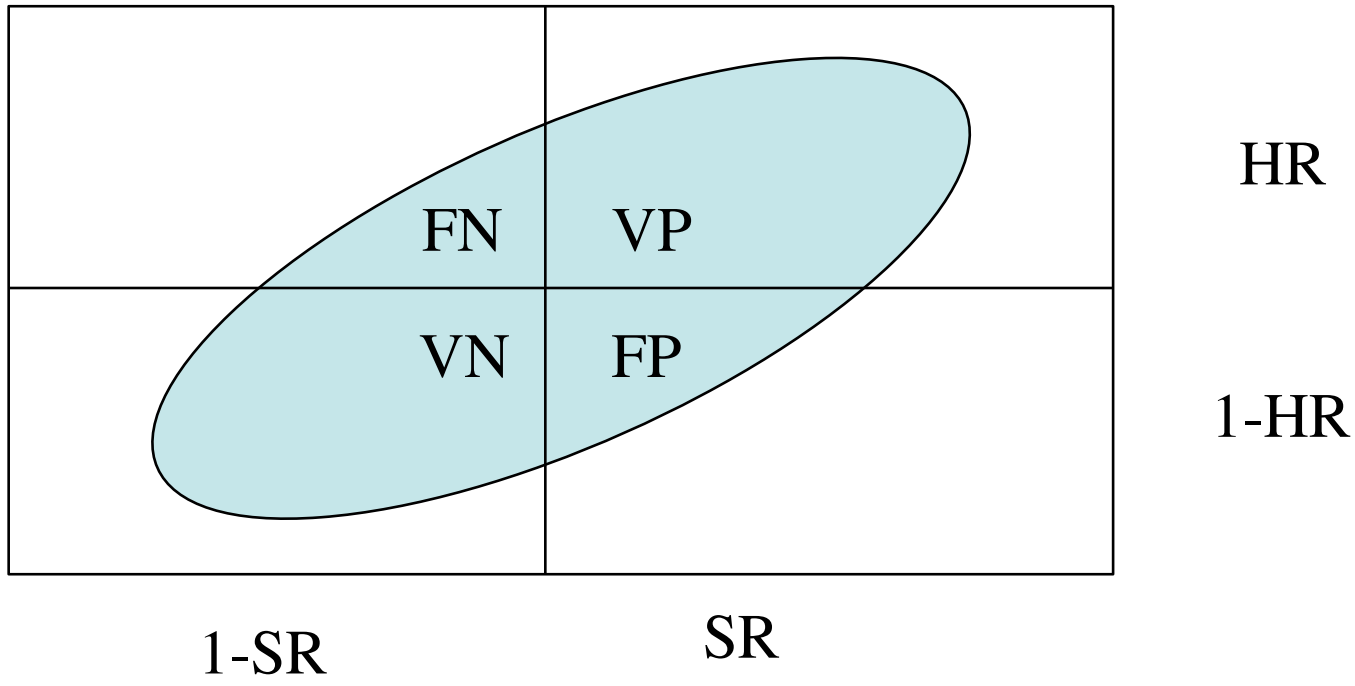
# Basic relationships

| | X | Y | Yp | Residual |
|---|---|---|---|---|
| Variance | $V_x$ | $V_y$ | $V_y(r^2)$ | $V_y(1-r^2)$ |
| Correl with X | 1 | $r_{xy}$ | 1 | 0 |
| Correl with Y | $r_{xy}$ | 1 | $r_{xy}$ | $\sqrt{(1-r^2)}$ |

# Phi coefficient of correlation

Hit Rate = Valid Positive + False Negative

Selection Ratio = Valid Positive + False Positive




Phi =(VP - HR*SR) /sqrt(HR*(1-HR)*(SR)*(1-SR)

# Correlation size ≠ causal importance

| | Pregnant | Not Pregnant | Total |
|---|---|---|---|
| Intercourse | 2 | 1,041 | 1,043 |
| No intercourse | 0 | 6,257 | 6,257 |
| Total | 2 | 7,298 | 7,300 |

# Correlation size ≠ causal importance

| | Pregnant | Not Pregnant | Total |
|---|---|---|---|
| Intercourse | 0.0003 | 0.1426 | 0.1429 |
| No intercourse | 0.0000 | 0.8571 | 0.8571 |
| Total | 0.0003 | 0.9997 | 1.0000 |

Phi =(VP - HR*SR) /sqrt(HR*(1-HR)*(SR)*(1-SR)= .04
polychoric rho = .53

# Sex discrimination?

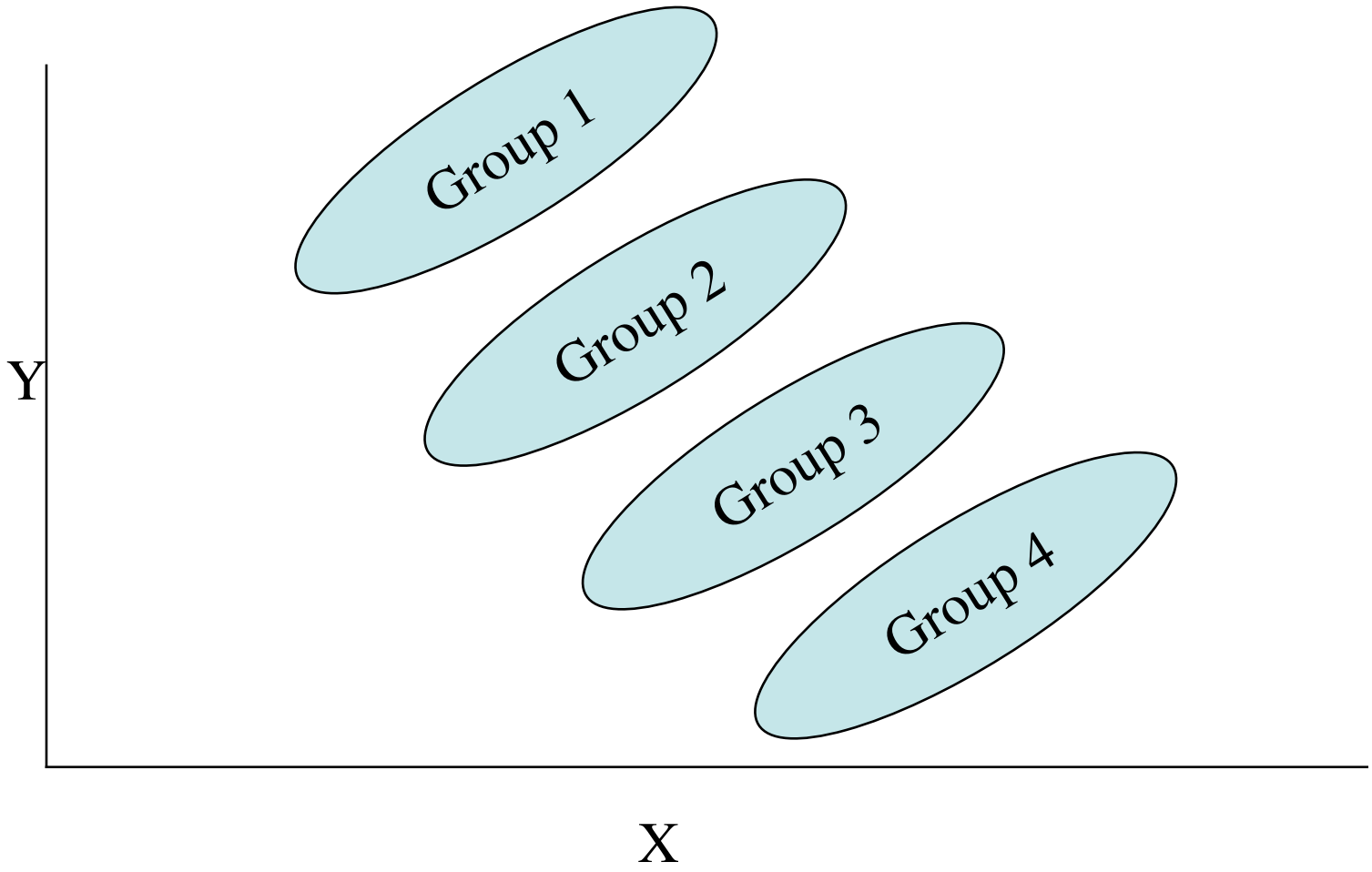
| | Admit | Reject | Total |
|---|---|---|---|
| Male | 40 | 10 | 50 |
| Female | 10 | 40 | 50 |
| Total | 50 | 50 | 100 |

Phi =(VP - HR*SR) /sqrt(HR*(1-HR)*(SR)*(1-SR)= -.60
polychoric rho = -.81

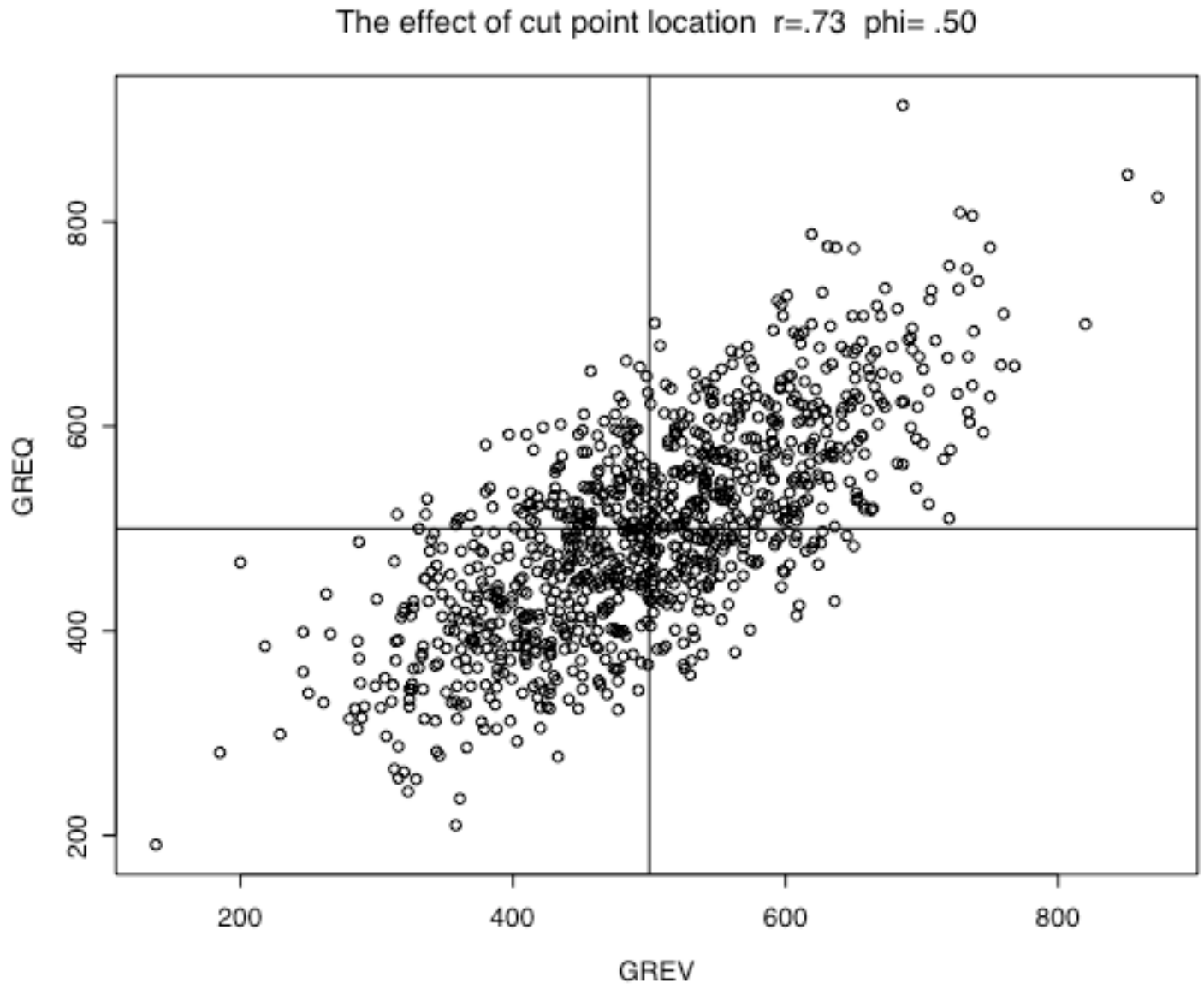
# Sex discrimination?

| | Department 1 | | | Department 2 | | |
|---|---|---|---|---|---|---|
| | Admit | Reject | Total | Admit | Reject | Total |
| Male | 40 | 5 | 45 | 0 | 5 | 5 |
| Female | 5 | 0 | 5 | 5 | 40 | 45 |
| Total | 45 | 5 | 50 | 5 | 45 | 50 |
| Phi | 0.11 | | | 0.11 | | |
| Pooled phi | | | -0.6 | | | |

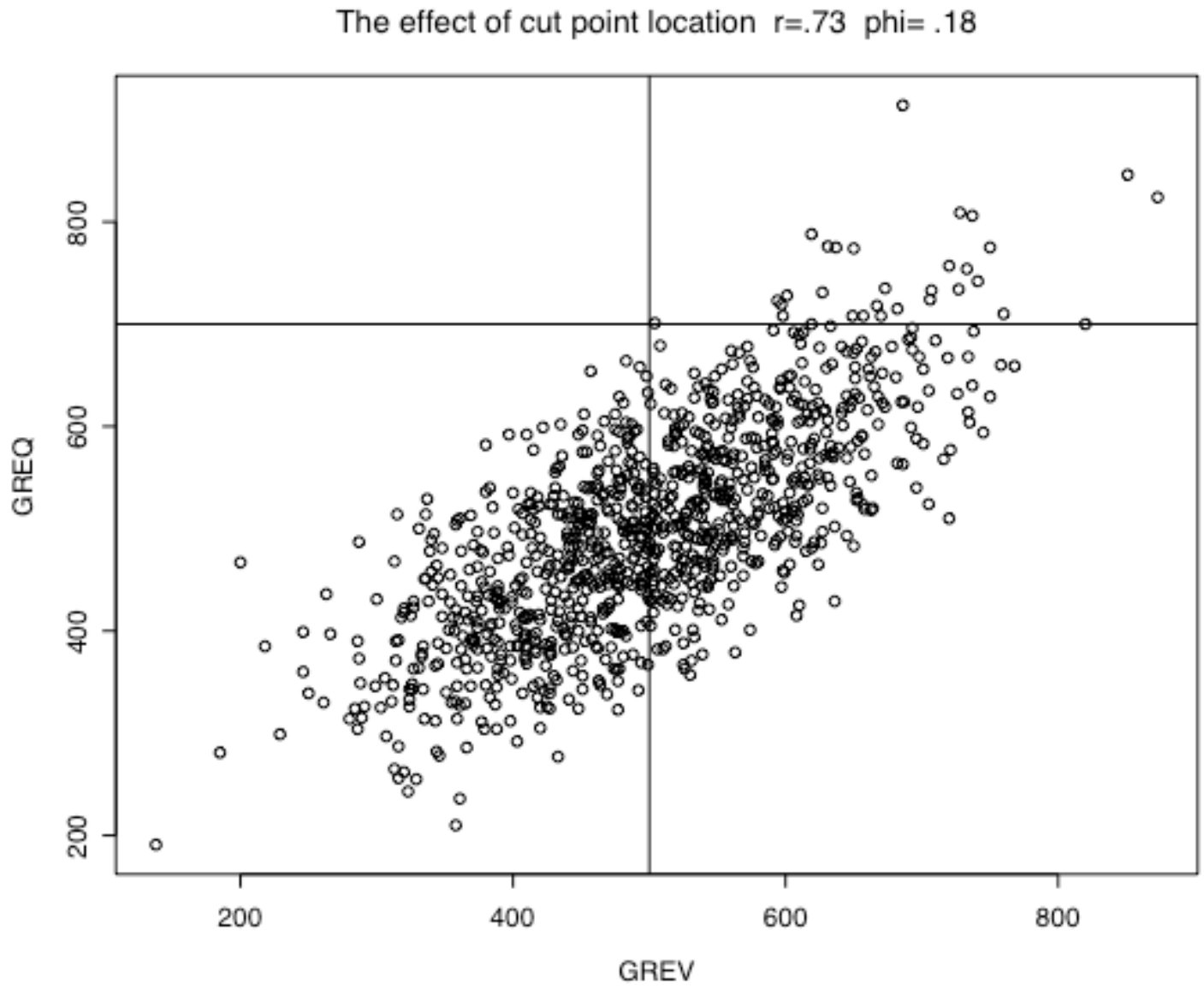
# Within group vs Between Group correlation



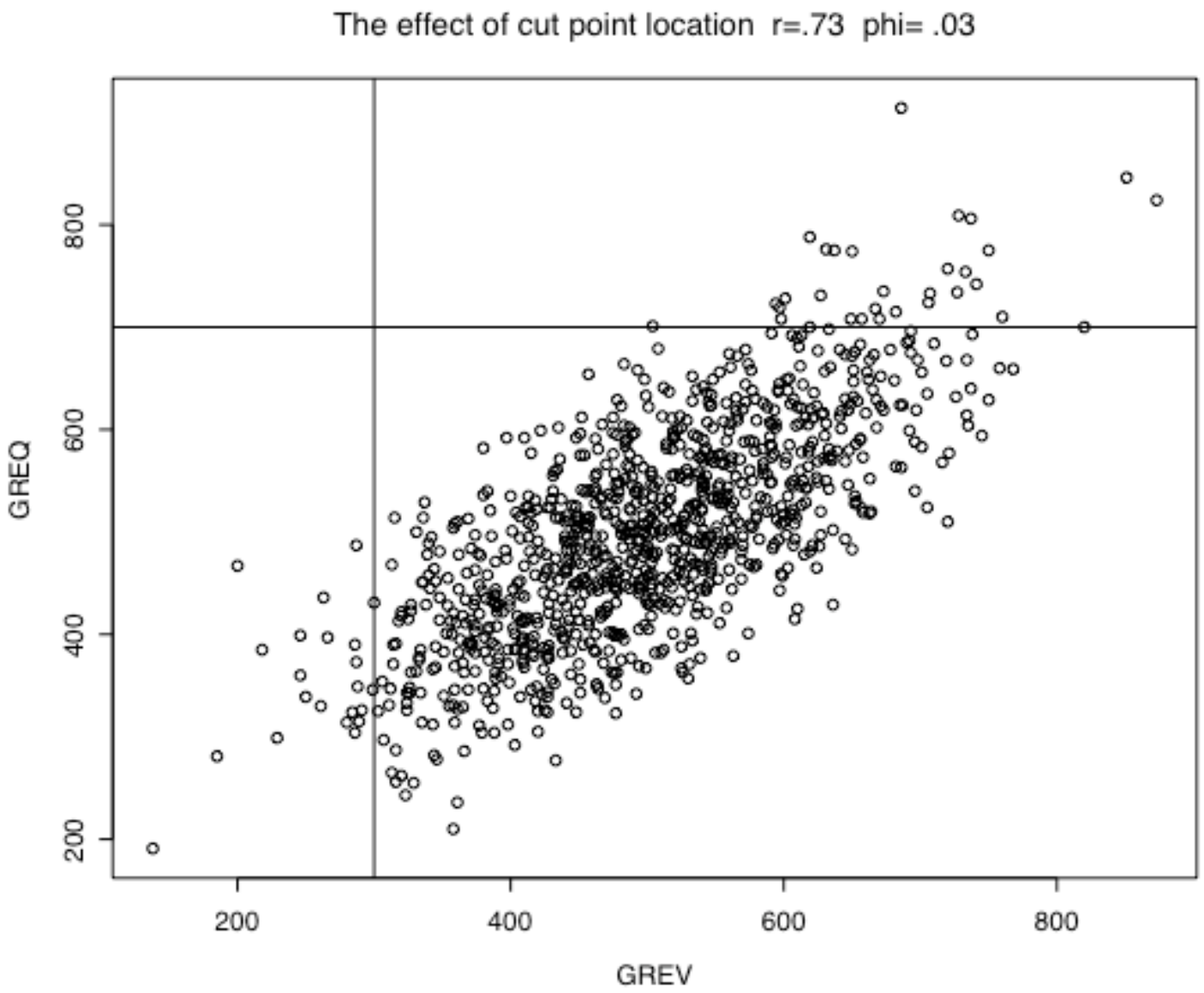
# Phi vs. r  the effect of cutpoints



# Phi vs. r the effect of cutpoints (2)



# Phi vs. r: extreme cutpoints



# Continuous and dichotomous scales

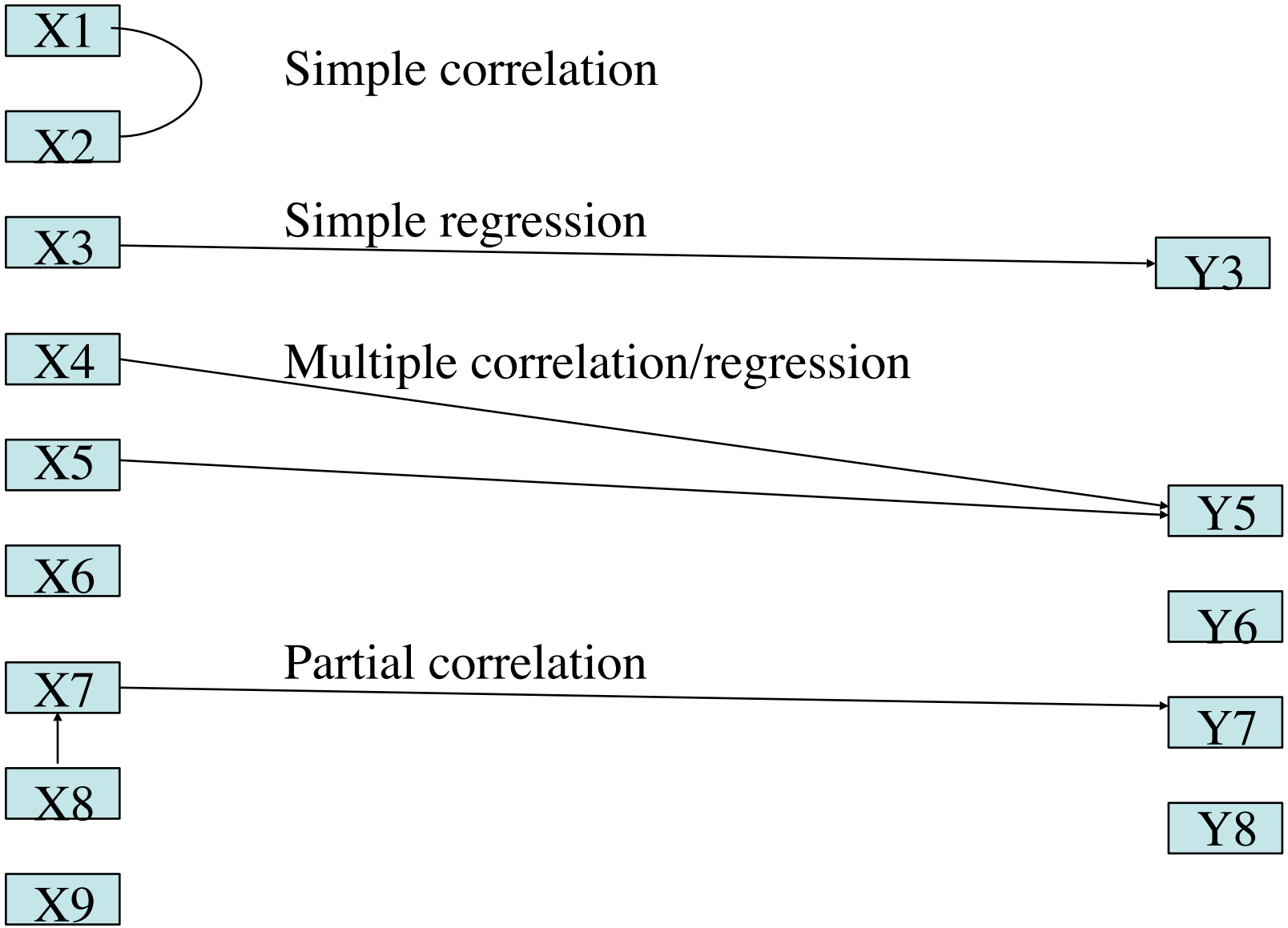
| | GREV | V2 | V2l | GREQ | Q2 | Q2h | GREA | GPA | MA |
|---|---|---|---|---|---|---|---|---|---|
| GREV | 1.00 | 0.80 | 0.34 | 0.73 | 0.57 | 0.30 | 0.64 | 0.42 | 0.32 |
| V2 | 0.80 | 1.00 | 0.15 | 0.58 | 0.50 | 0.18 | 0.51 | 0.37 | 0.23 |
| V2l | 0.34 | 0.15 | 1.00 | 0.21 | 0.15 | 0.03 | 0.19 | 0.15 | 0.12 |
| GREQ | 0.73 | 0.58 | 0.21 | 1.00 | 0.80 | 0.42 | 0.60 | 0.37 | 0.29 |
| Q2 | 0.57 | 0.50 | 0.15 | 0.80 | 1.00 | 0.18 | 0.45 | 0.29 | 0.21 |
| Q2h | 0.30 | 0.18 | 0.03 | 0.42 | 0.18 | 1.00 | 0.23 | 0.12 | 0.10 |
| GREA | 0.64 | 0.51 | 0.19 | 0.60 | 0.45 | 0.23 | 1.00 | 0.52 | 0.45 |
| GPA | 0.42 | 0.37 | 0.15 | 0.37 | 0.29 | 0.12 | 0.52 | 1.00 | 0.31 |
| MA | 0.32 | 0.23 | 0.12 | 0.29 | 0.21 | 0.10 | 0.45 | 0.31 | 1.00 |

V2, Q2 are cut at 500

V2l is cut at 300

Q2h is cut at 700

# Variance, Covariance, and Correlation

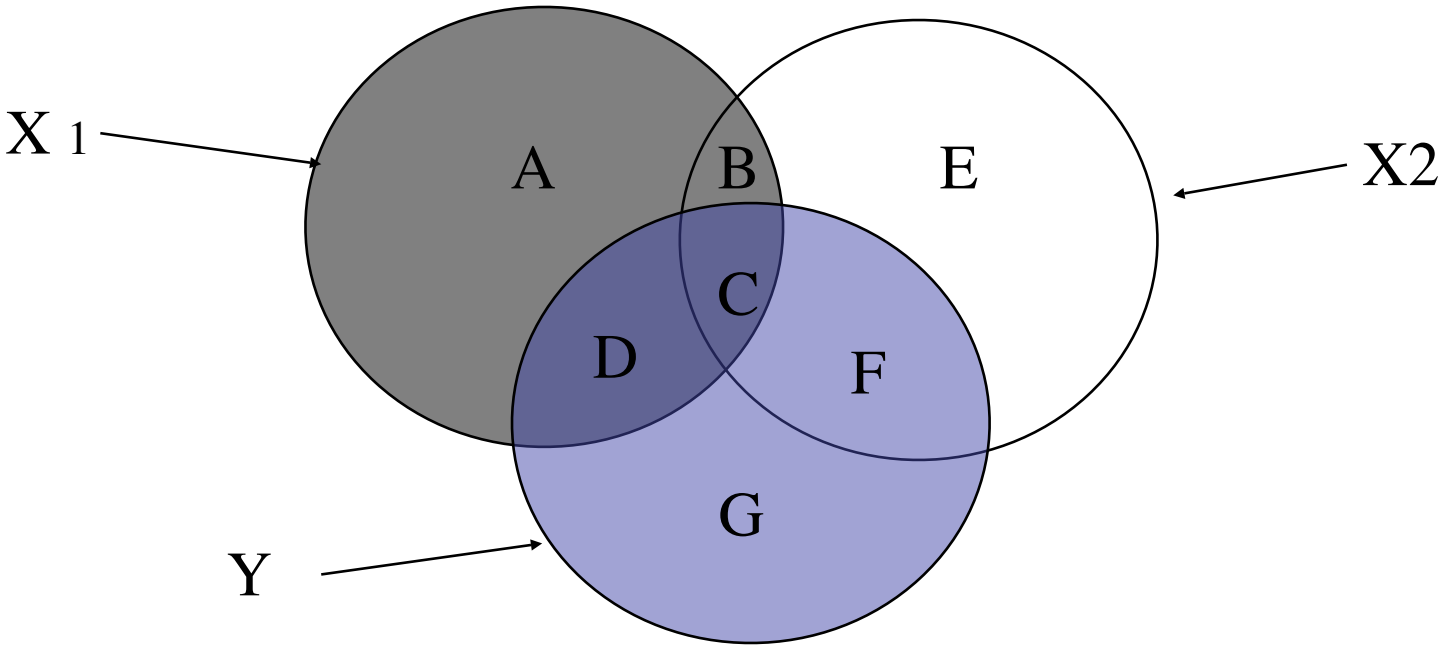


# Measures of relationships with more than 2 variables

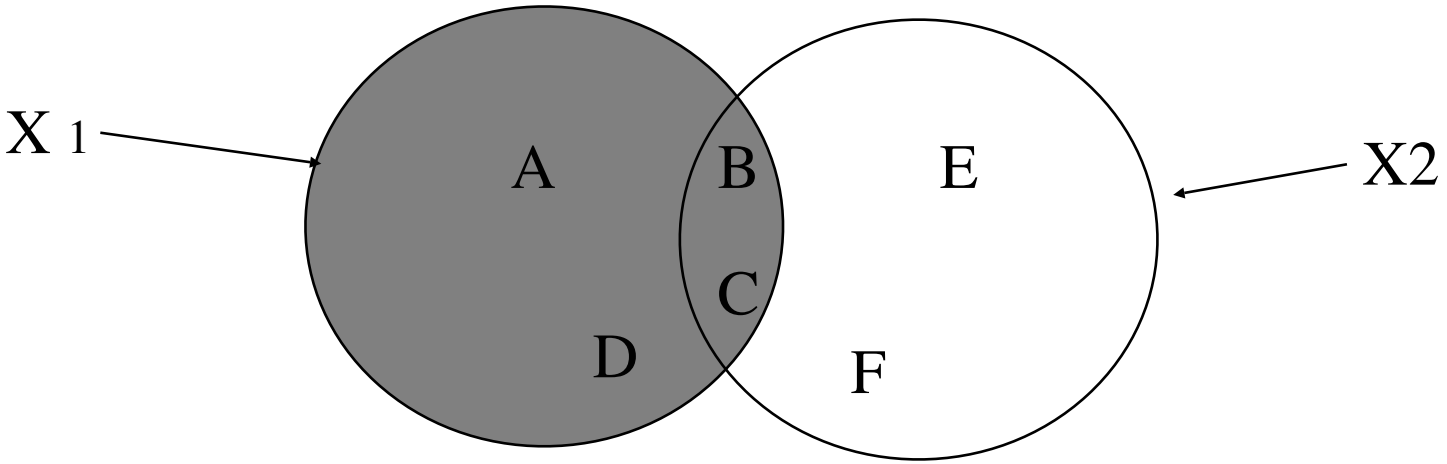
- Partial correlation
  - The relationship between x and y with z held constant (z removed)
- Multiple correlation
  - The relationship of x1 + x2 with y
  - Weight each variable by its independent contribution

# Partial and Multiple Correlation

The conceptual problem



# Variance, Covariance and Correlation




$V_1 = A + B + C + D$ $C_{12} = B + C$ $V_2 = E + B + C + F$

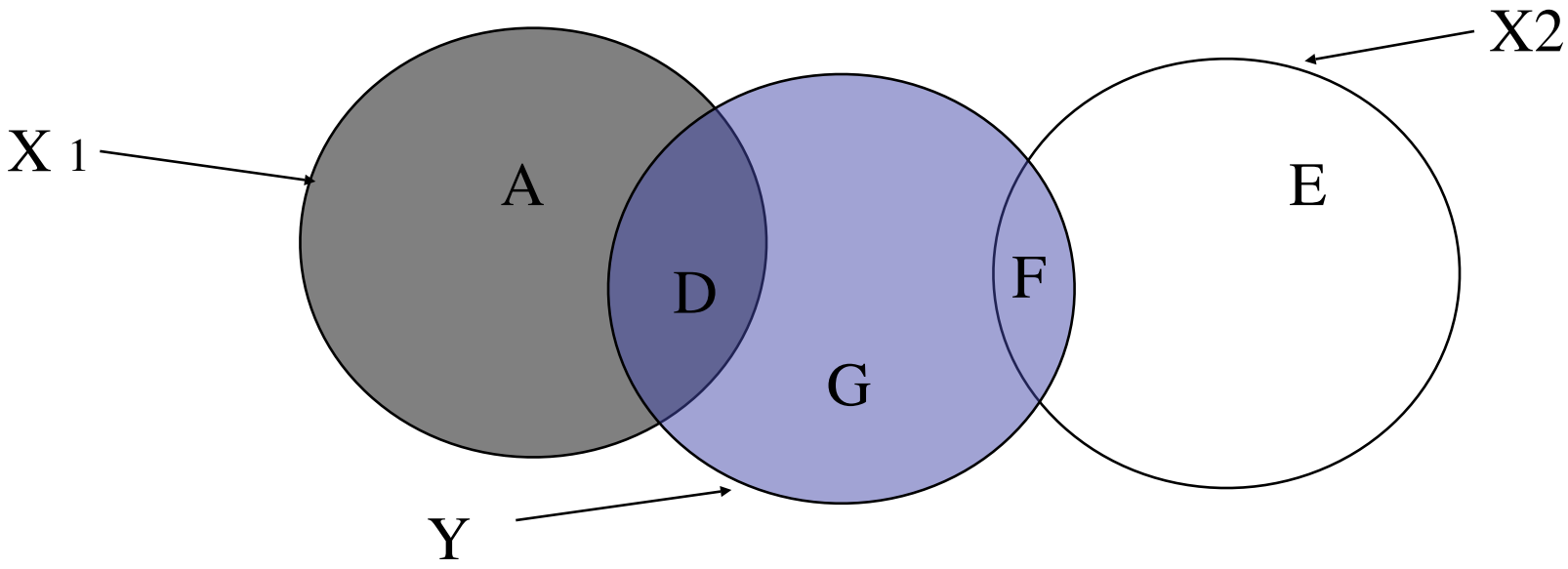
$r = C_{12}/\mathrm{sqrt}(V_1 V_2)$

$V_{1.2} = A + D$ $V_{2.1} = E + F$

$V_{1.2} = V_1(1-r^2)$ $V_{2.1} = V_2(1-r^2)$

# Multiple Correlation

## Independent Predictors




$V_1 = A + B + C + D$

$V_2 = E + B + C + F$

$V_Y = D + C + F + G$

$V_{1.2} = A + D$

$C_{12} = B + C$

$C_{1Y} = C + D$

$C_{2Y} = C + F$

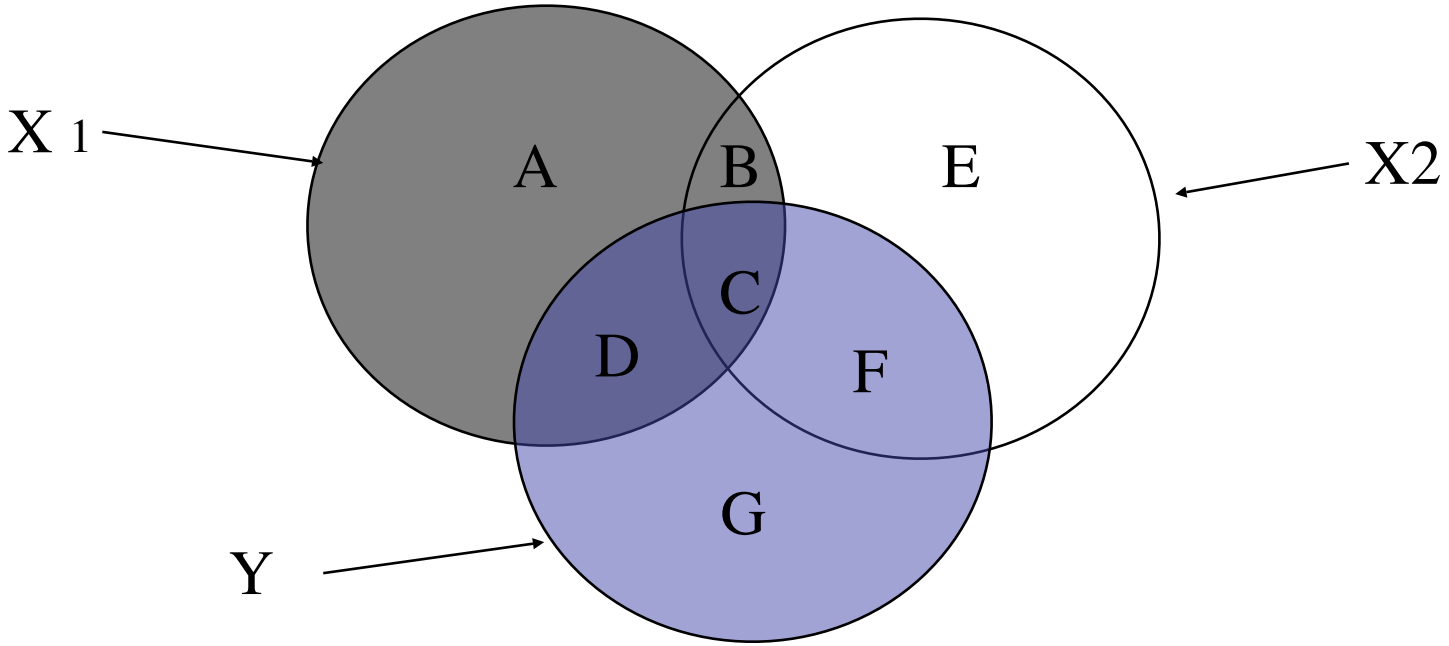
$V_{2.1} = E + F$

$C_{1Y.2} = D$

$C_{2Y.1} = F$

$C_{(12)Y} = D + C + F$

# Partial and Multiple Correlation




$V_1 = A + B + C + D$

$V_2 = E + B + C + F$

$V_Y = D + C + F + G$

$V_{1.2} = A + D$

$C_{12} = B + C$

$C_{1Y} = C + D$

$C_{2Y} = C + F$

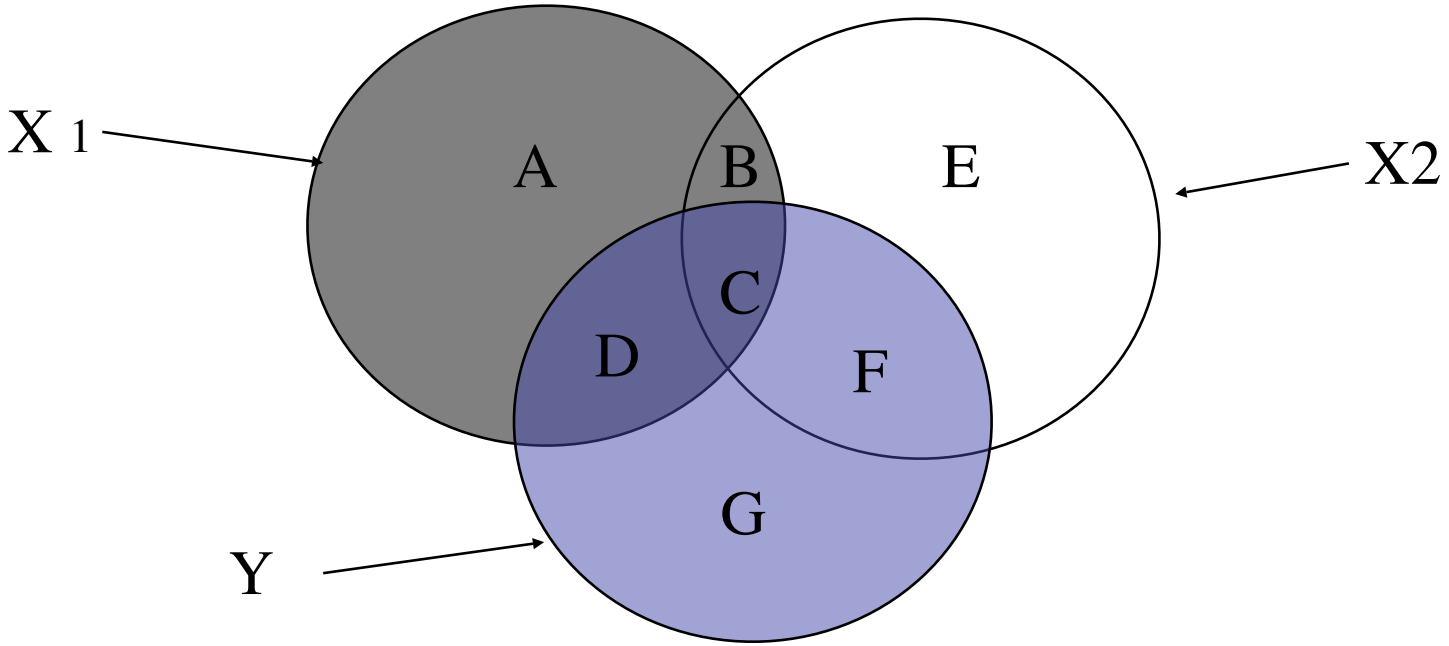
$V_{2.1} = E + F$

$C_{1Y.2} = D$

$C_{2Y.1} = F$

$C_{(12)Y} = D + C + F$

# Partial and Multiple Correlation: Partial Correlations




$V_1 = A + B + C + D$

$V_2 = E + B + C + F$

$V_Y = D + C + F + G$

$V_{1.2} = A + D$

$C_{12} = B + C$

$C_{1Y} = C + D$

$C_{2Y} = C + F$

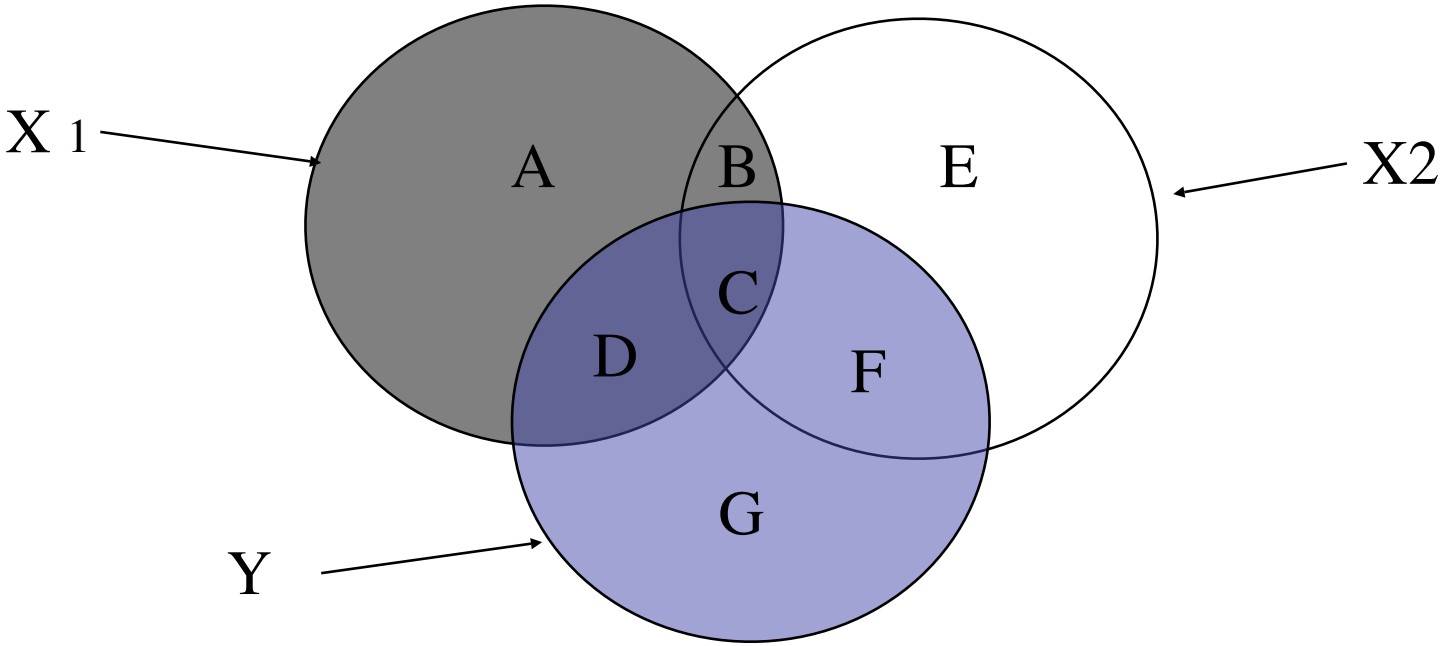
$V_{2.1} = E + F$

$C_{1Y.2} = D$

$C_{2Y.1} = F$

$$r_{1Y.2} = \frac{(r_{1y} - r_{12} * r_{2Y})}{\text{sqrt}((1 - r_{12}^2) * (1 - r_{y2}^2))}$$

# Partial and Multiple Correlation: Multiple Correlation-correlated predictors



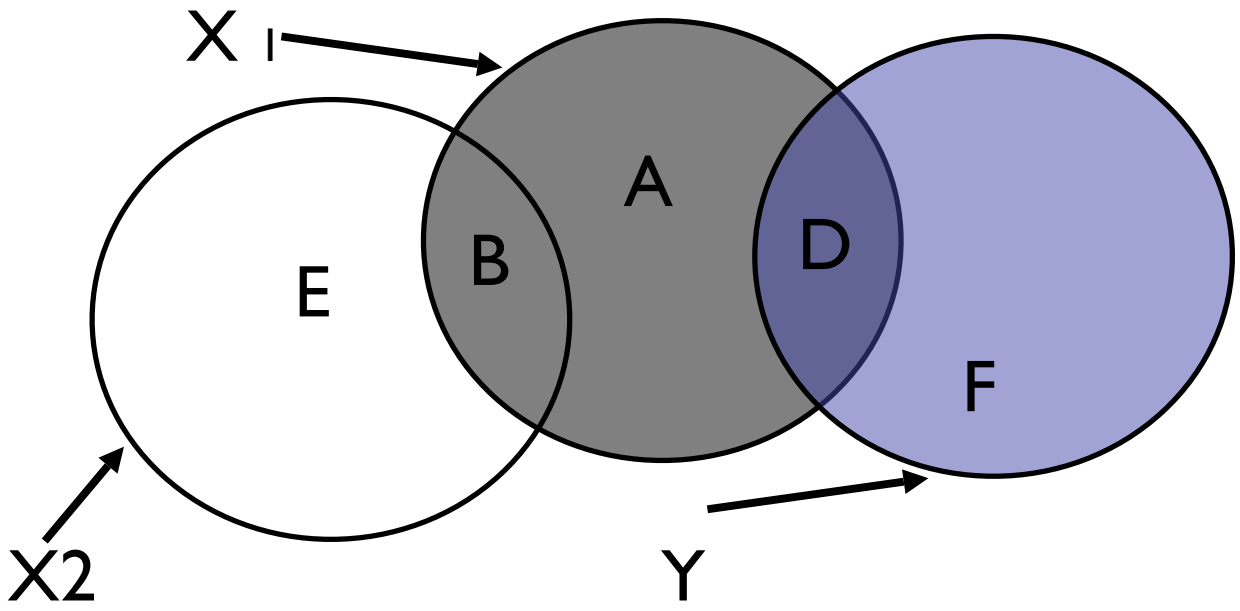

$Y = b_1X_1 + b_2X_2$

$b_1 = (r_{x1y} - r_{12}*r_{2y})/(1-r_{12}^2)$

$b_2 = (r_{x2y} - r_{12}*r_{1y})/(1-r_{12}^2)$

$R^2 = b_1r_1+b_2r_2$

# Multiple Correlation:




$V_1 = A + B + C + D$

$V_2 = E + B + C + F$

$V_Y = D + C + F + G$

$V_{1.2} = A + D$

$C_{12} = B + C$

$C_{1Y} = C + D$

$C_{2Y} = C + F$

$V_{2.1} = E + F$

$C_{1Y.2} = D$

$C_{2Y.1} = F$

$C_{(12)Y} = D + C + F$

# Multiple Correlation as an unweighted composite

| | $X_1$ | $X_2$ | $Y$ |
|---|---|---|---|
| $X_1$ | $Vx_1$ | $Cx_1x_2$ | $Cx_1y$ |
| $X_2$ | $Cx_1x_2$ | $Vx_2$ | $Cx_2y$ |
| $Y$ | $Cx_1y$ | $Cx_2y$ | $Vy$ |

$$Vx_1x_2 = Vx_1 + Vx_2 + 2Cx_1x_2$$

$$C(x_1x_2)y = Cx_1y + Cx_2y$$

$$R(x_1x_2)y = \frac{C(x_1x_2)y}{\text{Sqrt}(Vx_1x_2)*V_y}$$

# Multiple Correlation as a weighted composite

| | $b_1X_1$ | $b_2X_2$ | $Y$ |
|---|---|---|---|
| $b_1X_1$ | $b_1^2Vx_1$ | $b_1b_2Cx_1x_2$ | $b_1Cx_1y$ |
| $b_2X_2$ | $b_1b_2Cx_1x_2$ | $b_2^2Vx_2$ | $b_2Cx_2y$ |
| $Y$ | $b_1Cx_1y$ | $b_2Cx_2y$ | $Vy$ |

$$Vb_1x_1b_2x_2 = b_1^2Vx_1 + b_2^2Vx_2 + 2C\ b_1b_2Cx_1x_2$$

$$C(b_1x_1b_2x_2)y = b_1Cx_1y + b_2Cx_2y$$

$$R(b_1x_1b_2x_2)y = \frac{C(b_1x_1b_2x_2)}{Sqrt(Vb_1x_1b_2x_2)*V_y}$$

# Multiple Correlation as a weighted composite

| | $b_1X_1$ | $b_2X_2$ | Y |
|---|---|---|---|
| $b_1X_1$ | $b_1^2Vx_1$ | $b_1b_2Cx_1x_2$ | $b_1Cx_1y$ |
| $b_2X_2$ | $b_1b_2Cx_1x_2$ | $b_2^2Vx_2$ | $b_2Cx_2y$ |
| Y | $b_1Cx_1y$ | $b_2Cx_2y$ | Vy |

$$R(b_1x_1b_2x_2)y = \frac{C(b_1x_1b_2x_2)}{Sqrt(Vb_1x_1b_2x_2)*V_y}$$

Problem: Find b1, b2 to maximize R

$$b_1 = (r_{x1y} - r_{12}*r_{2y})/(1-r_{12}^2)$$

$$b_2 = (r_{x2y} - r_{12}*r_{1y})/(1-r_{12}^2)$$

# Multiple regression: Matrix approach

$$Y = X * b + e$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \dots \\ Y_i \\ \dots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & x_{21} & \dots & x_{k1} \\ 1 & x_{12} & x_{22} & \dots & x_{k2} \\ 1 & \dots & \dots & \dots & \dots \\ 1 & x_{1i} & x_{2i} & \dots & x_{ki} \\ 1 & \dots & \dots & \dots & \dots \\ 1 & x_{1n} & x_{2n} & \dots & x_{kn} \end{bmatrix} * \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \dots \\ b_k \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \dots \\ e_i \\ \dots \\ e_n \end{bmatrix}$$

# Matrix Algebra: a review

- Matrix algebra as a convenient notation for statistics
- Consider a matrix ${}_nX_m$ with n rows and m columns and elements $x_{ij}$
- Then X' (read X transpose) has m rows and n columns: ${}_mX'_n$ and elements $x_{ij}' = x_{ji}$
- ${}_mS_m = {}_mX'_n \, {}_nX_m$ is a m * m matrix of the sums (over n) of products with elements = $s_{ij} = \sum x_{ki} * x_{kj}$
- Note that if the number of columns = 1, then X is a vector with n rows. Then X'X = sum squares of x and XX' is a matrix of the products of x

# Matrix Algebra: a review (2)

- The identity matrix, ${}_nI_n$ has 1's on the diagonal and 0 elsewhere.

  $IX = XI = X$

- Matrix multiplication is associative but not commutative:

  $(XY)Z = X(YZ)$ but $XY \neq YX$

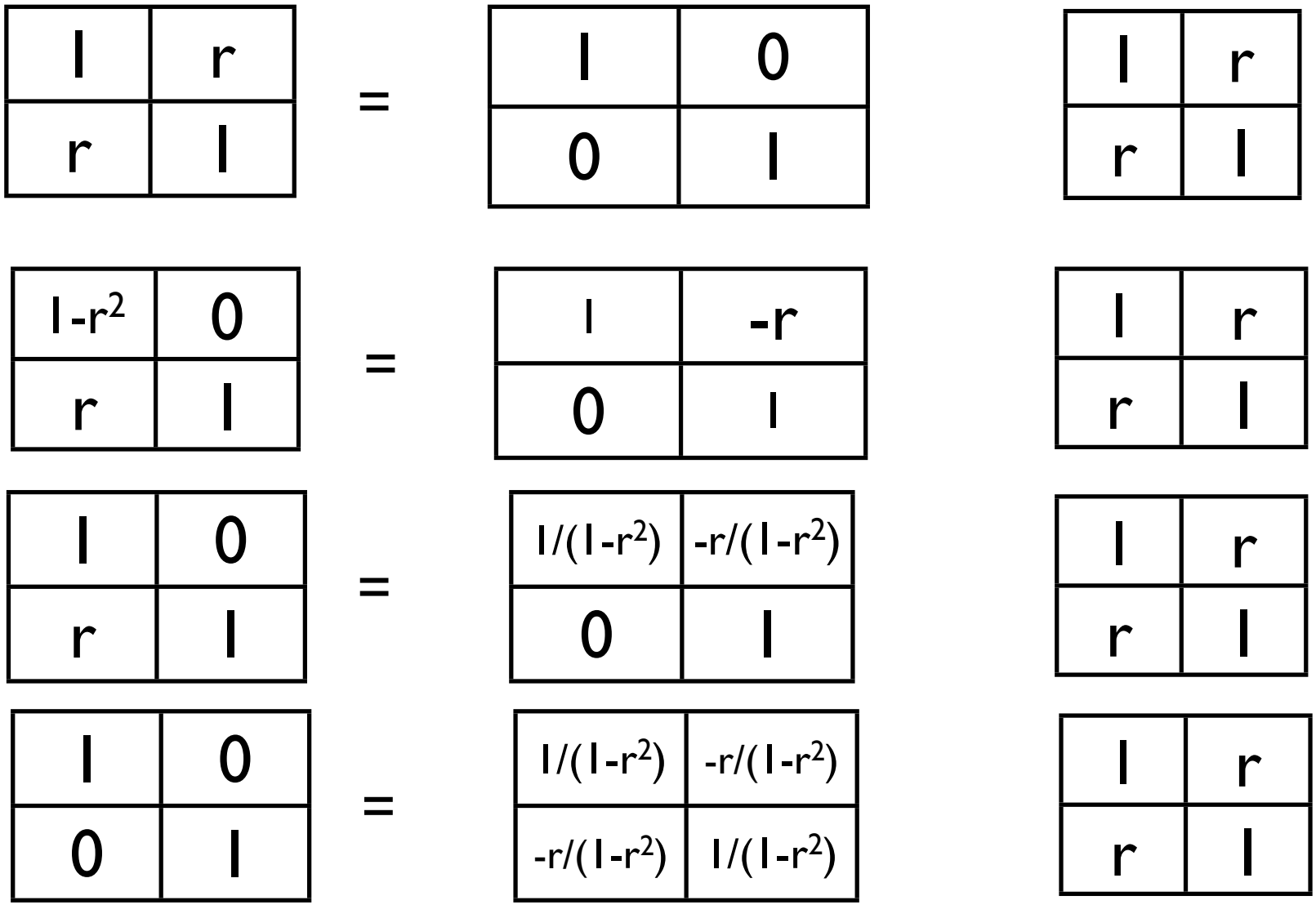
- For a square matrix, X, the inverse, $X^{-1}$ is that matrix, which when multiplied by X is I:

  $X^{-1} X = X X^{-1} = I$

# Matrix Algebra: a review (3)

- Finding the inverse $X^{-1}$ of X
- $X = IX$
- multiply both sides by a transformation with the goal of converting the left side to the Identity matrix:
  - $T_1X = T_1IX$
  - $T_2T_1X = T_2T_1IX$ until
  - $T_n \ldots T_2T_1X = I = T_n \ldots T_2T_1IX$ then
  - $(T_n \ldots T_2T_1)X = I \iff (T_n \ldots T_2T_1) = X^{-1}$

# finding the inverse



# Multiple regression: Matrix approach

- $Y = X * b + e$ (Y a vector, X a matrix)
- $X'Y = X'X\ b + X'e$
- $Cov_{xy} = R_{xx}b + Cov_{xe}$ (for standardized X, Y)
- Find that value of b that minimizes ||e||
- $b = (X'X)^{-1} * X'Y$
- $b = R^{-1}X'Y$
- If X is a vector, then this is what we have already found: $b = Cov_{xy}/Var_x$
- The multivariate case is thus just a generalization of the univariate case

# Multiple regression with matrix algebra (2)

$$X'Y = X'X\,b + X'e \qquad \begin{bmatrix} r_{yx} \\ r_{yz} \end{bmatrix} = \begin{bmatrix} 1 & r_{xz} \\ r_{xz} & 1 \end{bmatrix} \begin{bmatrix} b_{xy.z} \\ b_{zy.x} \end{bmatrix}$$

$$b = R^{-1}X'Y \qquad \begin{bmatrix} b_{xy.z} \\ b_{zy.x} \end{bmatrix} = \begin{bmatrix} 1/(1-r_{xz}^2) & -r_{xz}/(1-r_{xz}^2) \\ -r_{xz}/(1-r_{xz}^2) & 1/(1-r_{xz}^2) \end{bmatrix} \begin{bmatrix} r_{yx} \\ r_{yz} \end{bmatrix}$$

$$\begin{bmatrix} b_{xy.z} \\ b_{zy.x} \end{bmatrix} = \begin{bmatrix} (r_{yx}-r_{yz}\,r_{xz})/(1-r_{xz}^2) \\ (r_{yz}-r_{yx}\,r_{xz})/(1-r_{xz}^2) \end{bmatrix}$$

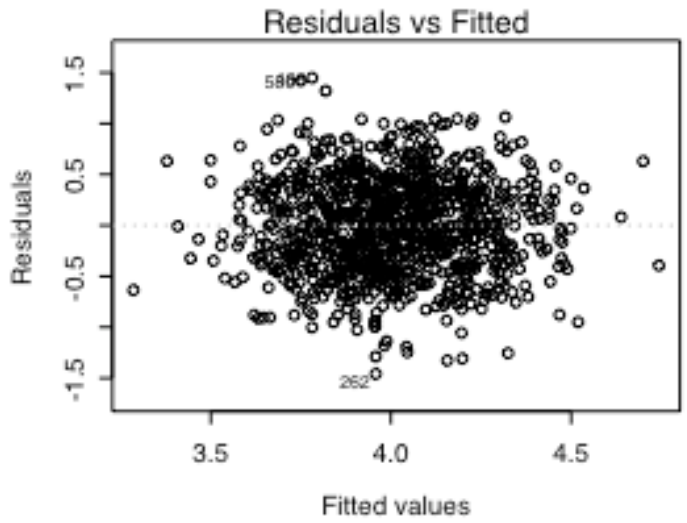
Compare this to the solution derived earlier

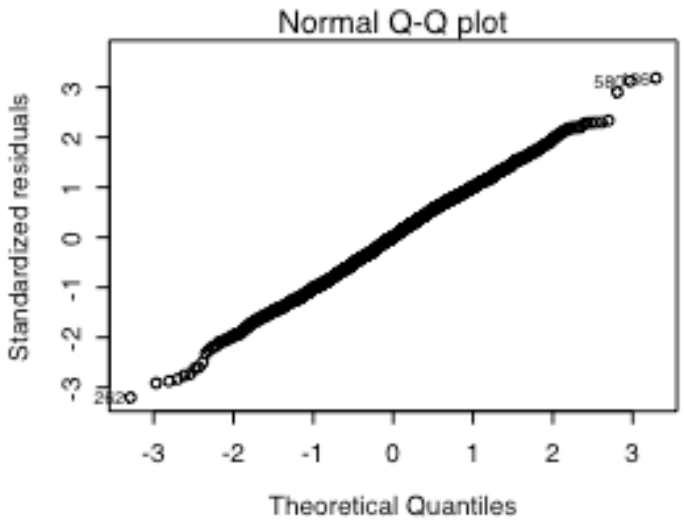
# Unit weights versus optimal weights - "It don't make no nevermind"

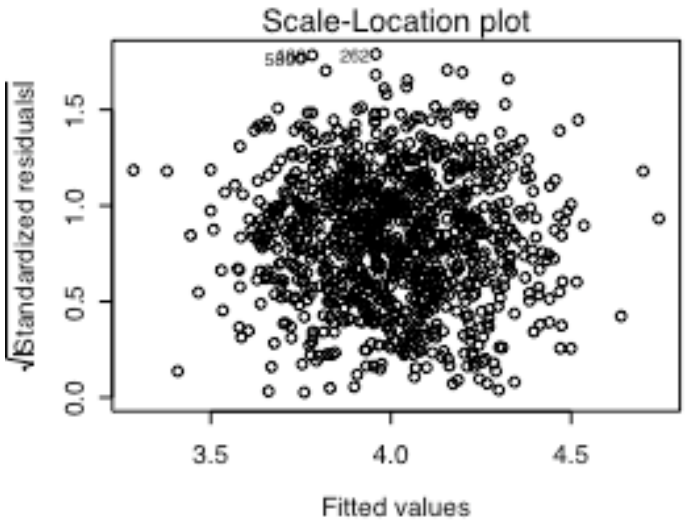
| $r_{x1x2}$ | $r_{x1y}$ | $r_{x2y}$ | beta 1 | beta 2 | R | $R^2$ | Unit Wt | $UW^2$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5 | 0.5 | 0.50 | 0.50 | 0.71 | 0.50 | 0.71 | 0.50 |
| 0.3 | 0.5 | 0.5 | 0.38 | 0.38 | 0.62 | 0.38 | 0.62 | 0.38 |
| 0.5 | 0.5 | 0.5 | 0.33 | 0.33 | 0.58 | 0.33 | 0.58 | 0.33 |
| 0.7 | 0.5 | 0.5 | 0.29 | 0.29 | 0.54 | 0.29 | 0.54 | 0.29 |
| 0.3 | 0.5 | 0 | 0.55 | -0.16 | 0.52 | 0.27 | 0.31 | 0.10 |
| 0.3 | 0.5 | 0.3 | 0.45 | 0.16 | 0.52 | 0.27 | 0.50 | 0.25 |

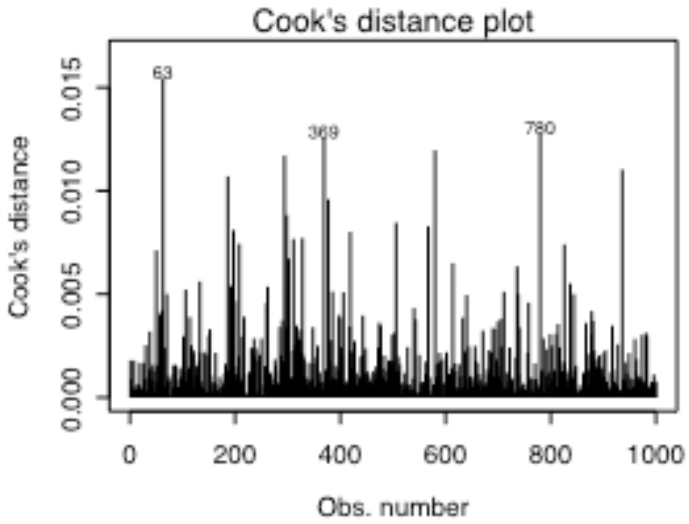
If $X_1$ and $X_2$ are both positively correlated with Y, then the effect of unit weighting versus optimal (beta) weighting is negligible. But, if one variable is not very good or zero,then unit weighting will not be as effective.

# Regression diagnostics



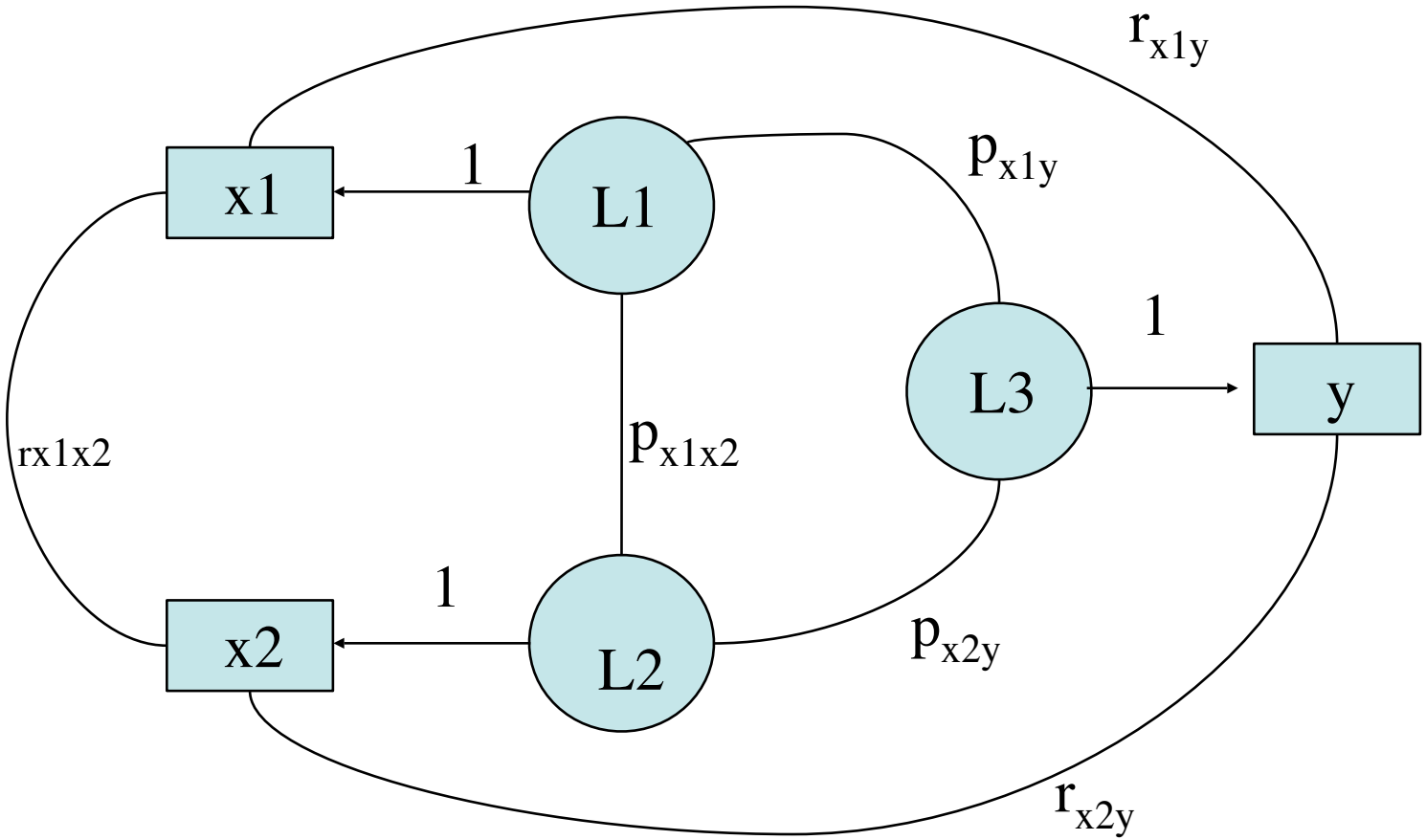



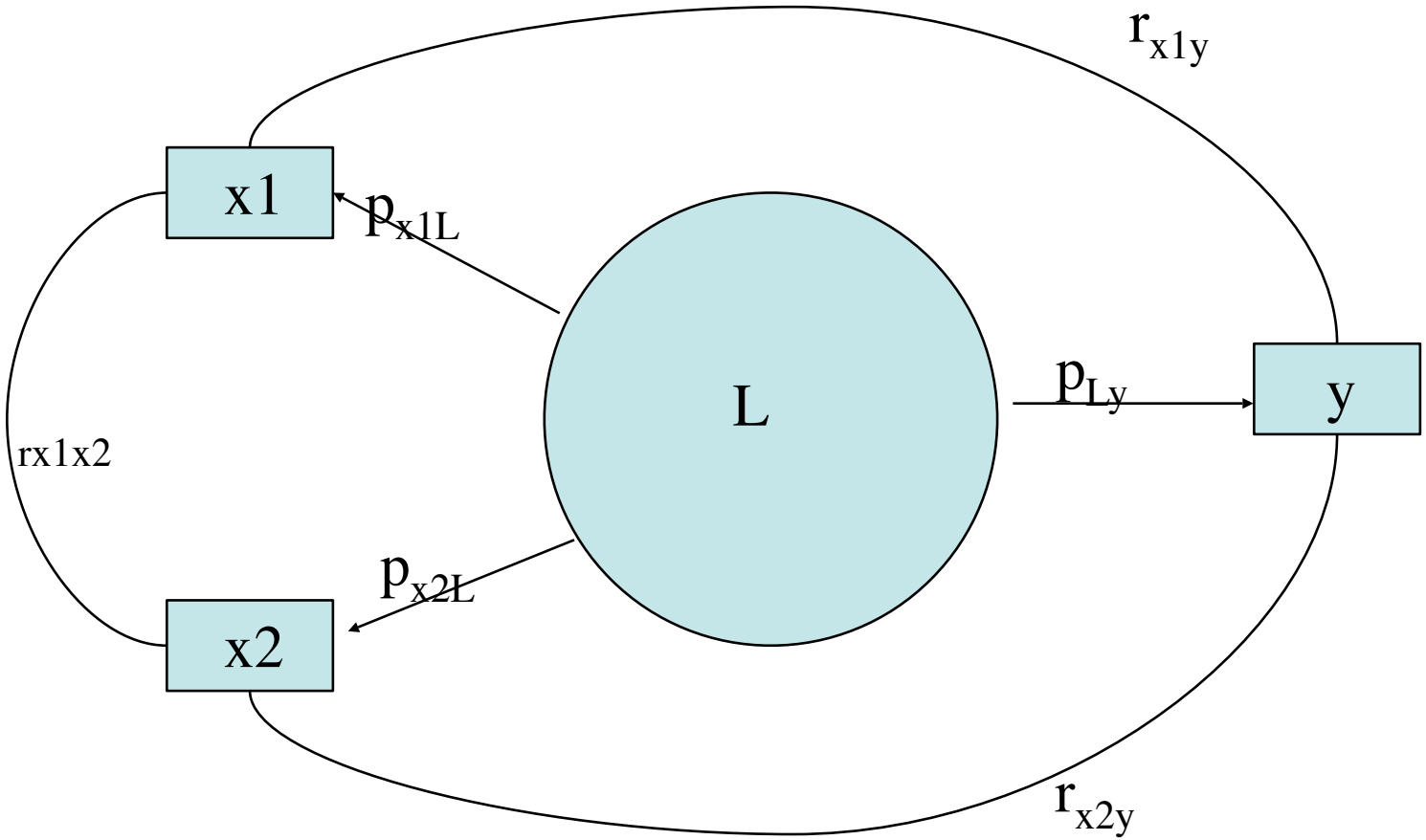
# Problems with correlations

- Simpson's paradox and the problem of aggregating groups
  - Within group relationships are not the same as between group or pooled relationships
- Phi coefficients and the problem of unequal marginal distributions
- Alternative interpretations of partial correlations

# Partial correlation: conventional model



# Partial correlation: Alternative model



# Partial Correlation: classical model

| | $X_1$ | $X_2$ | Y |
|---|---|---|---|
| $X_1$ | 1.00 | | |
| $X_2$ | 0.72 | 1.00 | |
| Y | 0.63 | 0.56 | 1.00 |

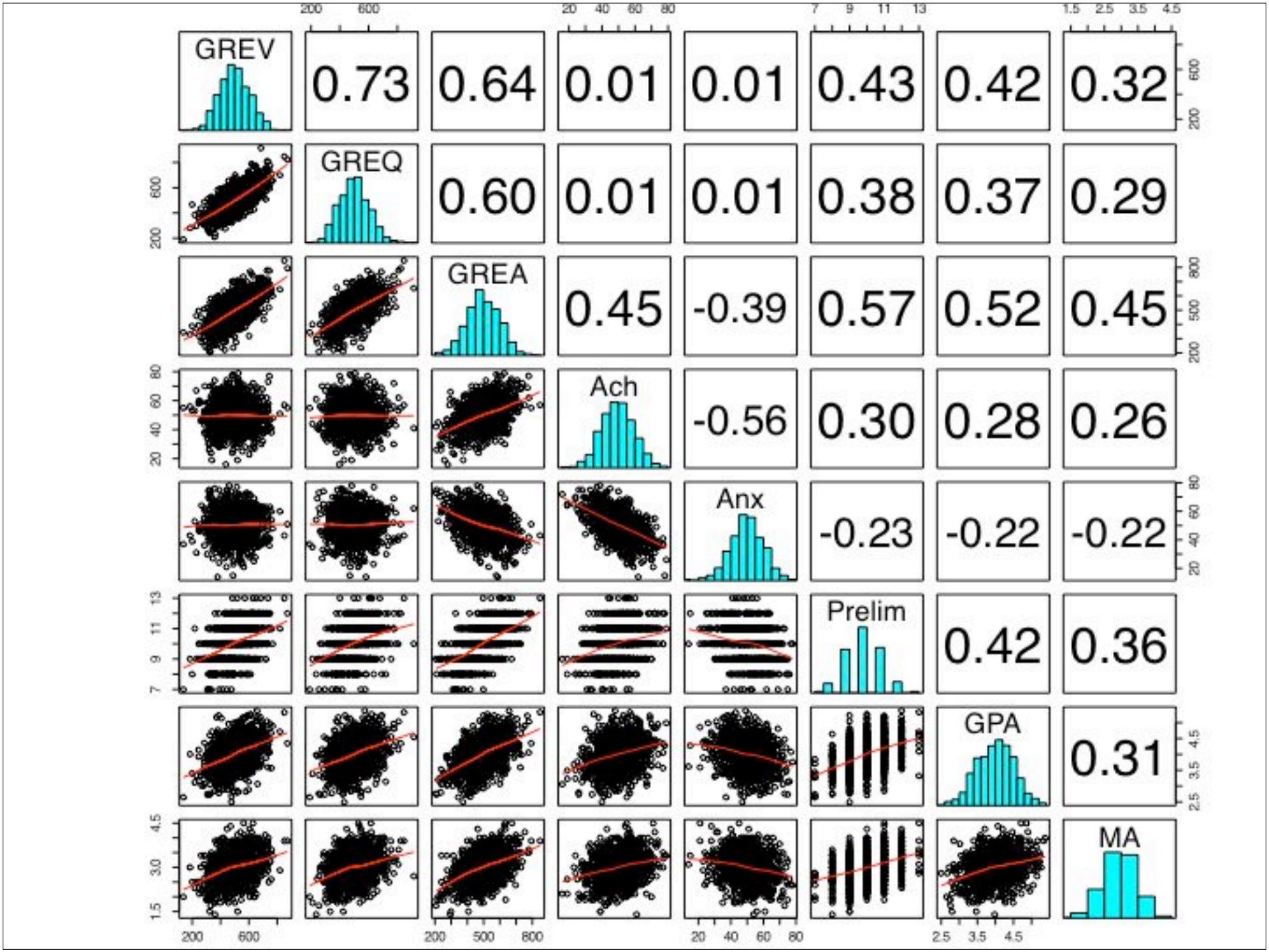
Partial r = $(r_{x1y}-r_{x1x2}*r_{x2y})/\mathrm{sqrt}((1-r_{x1x2}^2)*(1-r_{x2y}^2))$

Rx1y.x2 = .33 (traditional model) but = 0 with structural model

# Find the correlations

```
round(cor(dataset),2)          #find the correlation matrix
                               #round off to 2 decimals
```

| | GREV | GREQ | GREA | Ach | Anx | Prelim | GPA | MA |
|---|---|---|---|---|---|---|---|---|
| GREV | 1.00 | 0.73 | 0.64 | 0.01 | 0.01 | 0.43 | 0.42 | 0.32 |
| GREQ | 0.73 | 1.00 | 0.60 | 0.01 | 0.01 | 0.38 | 0.37 | 0.29 |
| GREA | 0.64 | 0.60 | 1.00 | 0.45 | -0.39 | 0.57 | 0.52 | 0.45 |
| Ach | 0.01 | 0.01 | 0.45 | 1.00 | -0.56 | 0.30 | 0.28 | 0.26 |
| Anx | 0.01 | 0.01 | -0.39 | -0.56 | 1.00 | -0.23 | -0.22 | -0.22 |
| Prelim | 0.43 | 0.38 | 0.57 | 0.30 | -0.23 | 1.00 | 0.42 | 0.36 |
| GPA | 0.42 | 0.37 | 0.52 | 0.28 | -0.22 | 0.42 | 1.00 | 0.31 |
| MA | 0.32 | 0.29 | 0.45 | 0.26 | -0.22 | 0.36 | 0.31 | 1.00 |


GREV
GREQ
GREA
Ach
Anx
Prelim
GPA
MA
0.73
0.64
0.01
0.01
0.43
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0.32
0.60
0.01
0.01
0.38
0.37
0.29
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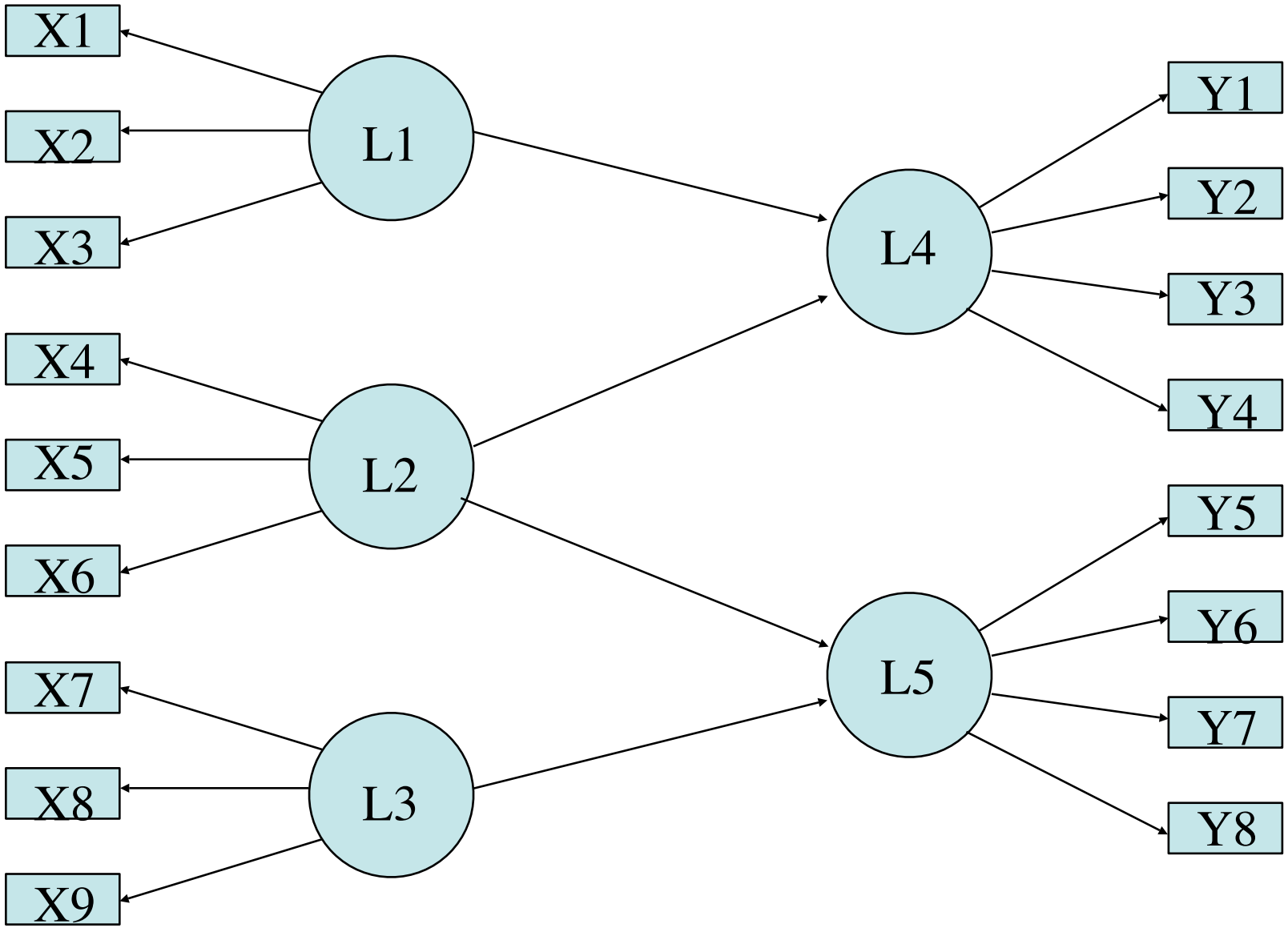
-0.39
0.57
0.52
0.45
-0.56
0.30
0.28
0.26
-0.23
-0.22
-0.22
0.42
0.36
0.31

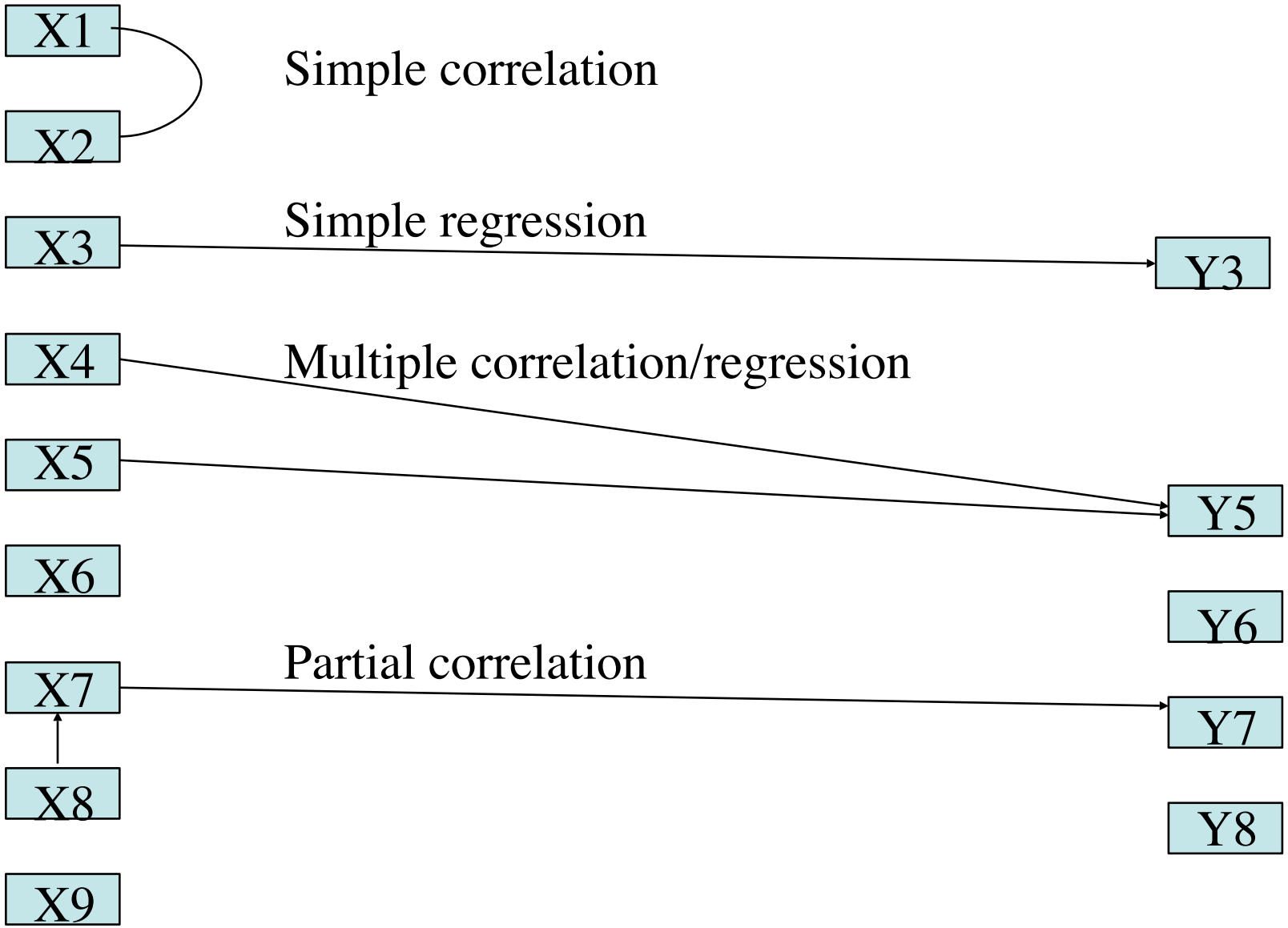
# Psychometric Theory: A conceptual Syllabus



# Measures of relationship

- Regression $y = bx + c$
  - $b_{y.x} = Cov_{xy} / Var_{x}$
- Correlation
  - $r_{xy} = Cov_{xy} / sqrt(V_x * V_y)$
  - Pearson Product moment correlation
    - Spearman (ppmc on ranks)
    - Point biserial (x is dichotomous, y continuous)
    - Phi (x, y both dichotomous)

# Variance, Covariance, and Correlation



# Measures of relationships with more than 2 variables

- Partial correlation
  - The relationship between x and y with z held constant (z removed)
- Multiple correlation
  - The relationship of x1 + x2 with y
  - Weight each variable by its independent contribution

# Problems with correlations

- Simpson's paradox and the problem of aggregating groups
  - Within group relationships are not the same as between group or pooled relationships
- Phi coefficients and the problem of unequal marginals
- Alternative interpretations of partial correlations